The Indian Economic Journal

JOURNAL OF THE INDIAN ECONOMIC ASSOCIATION

Special Issue, January 2022

Balanced Regional Development

- Disparities in Economic Development
- Social Development across Regions
- Poverty and Inequality Disparities
- 15th Finance Commission Recommendations and Regional Development
- Regional Pattern of Employment and Migration
- Infrastructure Development across Regions
- Case Studies of Selected States Including Intra-State Disparities
- Public Policies for Reduction of Regional Disparities

Editor's Message

The Indian Economic Journal (IEJ) is an important organ of the Indian Economic Association, provides support and services to professionals and researchers both in India and abroad. The Indian Economic Association is the largest and oldest body of teachers, researchers, academicians as well as policy makers drawn from the background of Economics and its affiliated disciplines, exhibited outstanding record of achievements over a century. Formed in 1917, the Indian Economic Association has been a scholarly, non-political, and non-profit oriented professional association open to all persons as per the eligibility criteria laid by the Constitution. It seeks through Conferences, Courses, publications and seminars to enable contact and dissemination of information among scholars to increase their understanding of economics. Both IEJ and IEA work in tandem motivating members to contribute articles in Annual Conferences and publish in special issues of the IEJ by maintaining relevance of the journal.

The Indian Economic Journal was founded by Prof. C.N. Vakil and Prof. R. Balakrishna in 1953 and it became the internationally acclaimed journals in Economics because of the hard work put in by successive editors in the management process of the over the years. The IEJ is at present included in the 'Abstract Services' of American Economic Association through their Journal of Economic Literature. I take this opportunity to acknowledge the contributions of Prof. Sukhdev Thorat in transforming Conference Volumes of IEA into special issues of Indian Economic Journal and Dr. Anil Kumar Thakur, Chief Convener of IEA for taking efforts to sustain the quality and ratings of IEJ along with the Managing Editor of IEA, Prof. Sudhanshu Bhushan.

The overreaching theme of the 103rd Annual conference is "Accelerating Economic Growth, Balanced Regional Development and Sustainable Urbanization in the aftermath of COVID-19". The sub themes are: 1. Accelerating Economic Growth: Trends and Way Forward 2. Sustainable Urbanization; 3. Banking and Financial Sector for New India; 4. Balanced Regional Development; and 5. Human Resource Development in The Context of New Technological Revolution. The articles in the volumes of the special issue of IEJ are the selected papers from each sub-theme.

I would like to thank the contributions of various authors by submitting papers for the 103rd Annual Conference and apologies for keeping them waiting for long due to the Covid-19 Pandemic and the subsequent postponement of the Annual Conference to be held in 2020. There may be lapses on my part on attending and commenting on queries routinely raised by authors for a year or more. I again seek your pardon for any lapse.

I also express my gratitude to Prof. Satyapriya Hiralal Indurwade, Prof. Kanhaiya Ahuja, Dr. Sandhya Rani Das, Prof. G M Bhatt and Dr. Anup Kumar for their help in reviewing the papers. I also thank all the authors, reviewers and the editorial supports, especially the services extended by Dr.P. Anbalagan, Dr. N. Suresh Babu, Dr. M. Dillip Anand, Dr. Kumari Manisha and Dr. A. Thaha Sahad in bringing out all volumes of the special issues of IEJ in scheduled time. Despite taking utmost care in compiling the volumes, if appears there any inadvertent lapses, I take the full responsibility and apologise.

Last but not the least, my sincere thanks to Er. A. Aashik Ahamed, TAMCOS Ltd, Chennai and his team for their nice execution and printing of the work in time.

B.P. Chandramohan

THE INDIAN ECONOMIC ASSOCIATION
Special Issue, Conference 2022

CONTENTS

A Note on The Use of A Measure of Economic Inequality in Analogue to The Refractive Index of Geometrical Optics

Amlan Majumder

ABSTRACT:

Refractive index of geometrical optics is found to be a good measure of segment-wise analysis of economic inequality under the Lorenz curve framework. For a distribution, when all such segment-wise index values are added, it becomes equivalent to the inequality measures based on the length of the Lorenz curve, which is pro-transfer sensitive, and by definition, additively decomposable. The analogy of refraction of light and bending of Lorenz curve is simple. While propagation, as a ray of light refracts according to characteristics of different media, so also Lorenz curve does according to concentration of wealth or income in different strata. Some studies in recent past erroneously considered the proposed index as an extension of Gini coefficient, and overlooked its visual appeal through Lorenz curve framework and that of refraction of light. The sole objective of this paper is to rediscover the index to make its appeal clear.

Key words: Additively decomposable, Amato-Kakwani inequality index, Length of the Lorenz curve, Pro transfer-sensitive property, Refractive inequality index

JEL index: D63, C88, Y80

INTRODUCTION

A composite measure of economic inequality, primarily which was derived in analogue to the index of refraction of geometrical optics to indicate inequality condition in each income group in contrast to the ideal condition, and the summation of which over n number of observations or groups finally appeared equivalent to the summary measures of inequality based on the length of the Lorenz curve, appeared initially in Majumder (2014) and in a more modified form in Majumder (2015). As a ray of light bents or gets refracted while propagation according to characteristics of different media, so also a Lorenz curve does while passing from one stratum into another according to concentration of wealth or income in different strata. The extent of bending of a ray of light, in case of a discrete approach, is measured by Snell's Law, which is a subject of optics or physical science and was discussed widely in standard text books of physics or optics. The index that measures the extent of bending of a ray of light is known as index of refraction or refractive index. When the values of refractive index corresponding to all the strata under a Lorenz curve framework are added and standardised, the index of overall inequality becomes equivalent to the inequality measures proposed by Amato (1968, p. 261), Lombardo (1969).

Department of Economics, University of North Bengal, P. O. North Bengal University
Dist. Darjeeling, West Bengal, India

measures based on the length of the Lorenz curve, may be used to study [illegible] along different segments of the Lorenz curve as well as for the whole framework [illegible]

Declarations

Funding: The author did not receive support from any organisation for the subm[illegible]

Conflicts of interest/Competing interests: (i) The author has no relevant [illegible] financial interests to disclose. (ii) This article does not contain any studies with [illegible] or animals performed by the author.

Availability of data and material: The datasets generated during and/or [illegible] current study are available in the UNU-WIDER [illegible] https://www.wider.unu.edu/project/wiid-%E2%80%93-world-income-inequality-[illegible]

Code availability: Relevant SPSS (15.0) command codes are presented in Annex[illegible]

Acknowledgement: The author is thankful to all for comments and suggestions. [illegible] any error rests with the author.

References

Amato, V., Metodologia Staistica Strutturale vol.1, Cacucci, Bari, 1968.

Anand, S., Inequality and Poverty in Malaysia: Measurement and Decomp[illegible] University Press, New York, 1983.

Arnold, B.C., "Inequality Measures for Multivariate Distributions," METRON - [illegible] Journal of Statistics, 63 (3), 317-327, 2005.

--------------., "On the Amato inequality index," Statistics and Probability Letters, [illegible] 1506, 2012.

Atkinson, A.B., "On the measurement of inequality," Journal of Economic Theory, [illegible] 1970.

Bellù, L. G. and Liberati, P., Inequality and Axioms for its Measurement, Food and [illegible] Organisation of the United Nations, Rome, 2006 (Retrieved August, 11, [illegible] http://www.fao.org/policy-support/tools-and-publications/resources-details/en/c/848[illegible]

Chateauneuf, A. and Gajdos, T., & Wilthien, P-H., "The Principle of Strong [illegible] Transfer," Journal of Economic Theory, 103 (2), 311-333, 2002.

Feynman, R. P., Feynman Lectures on Physics 1: Mainly Mechanics, Radiation, and [illegible] Books, New York, 2011.

Jenkins, A. F. and White, H. E., Fundamentals of Optics, McGrawhill, New Delhi, [illegible]

The Indian Economic Journal

REGISTERED WITH THE REGISTRAR
OF NEWSPAPER FOR INDIA
RNI Regn.No. 46913/87

ISSN 0019-4662

9 770019 466005 >

Majumder, A. (2022). “A note on the use of a measure of economic inequality in analogue to the refractive index of geometrical optics”, Indian Economic Journal, Special Issue/Conference Volume of 103rd Indian Economic Association of 2020 (January 2022): 323-345 (ISSN: 0019-4662).

A note on the use of a measure of economic inequality in analogue to the refractive index of geometrical optics

Amlan Majumder
Department of Economics
University of North Bengal
Raja Rammohunpur, P. O. North Bengal University
DT. Darjeeling, Pin. 734 013
West Bengal, India
Email: amlan@amlan.co.in

Abstract: Refractive index of geometrical optics is found to be a good measure of segment-wise analysis of economic inequality under the Lorenz curve framework. For a distribution, when all such segment-wise index values are added, it becomes equivalent to the inequality measures based on the length of the Lorenz curve, which is pro-transfer sensitive, and by definition, additively decomposable. The analogy of refraction of light and bending of Lorenz curve is simple. While propagation, as a ray of light refracts according to characteristics of different media, so also Lorenz curve does according to concentration of wealth or income in different strata. Some studies in recent past unknowingly considered the proposed index as an extension of Gini coefficient, and overlooked its visual appeal through Lorenz curve framework and that of refraction of light. The sole objective of this paper is to rediscover the index to make its appeal clear.

Key words: Additively decomposable, Amato-Kakwani inequality index, Length of the Lorenz curve, Pro transfer-sensitive property, Refractive inequality index

JEL codes: D63, C88, Y80

1. Introduction

A composite measure of economic inequality, primarily which was derived in analogue to the index of refraction of geometrical optics to indicate inequality condition in each income group in contrast to the ideal condition, and the summation of which over n number of observations or groups finally appeared equivalent to the summary measures of inequality based on the length of the Lorenz curve, appeared initially in Majumder (2014) and in a more modified form in Majumder (2015). As a ray of light bents or gets refracted while propagation according to characteristics of different media, so also a Lorenz curve does while passing from one stratum into another according to concentration of wealth or income in different strata. The extent of bending of a ray of light, in case of a discrete approach, is measured by Snell's Law, which is a subject of study in physical science and was discussed widely in standard text books of physics or optics. The index that measures the extent of bending of a ray of light is known as index of refraction or refractive index. When the values of refractive index corresponding to all the strata under a Lorenz curve framework are added and standardised, the index of overall inequality becomes equivalent to the inequality measures proposed by Amato (1968, p. 261), Lombardo (1969), Scala (1969) and Kakwani (1980a, pp. 83-85), which were discussed adequately, among other, by Arnold (2005, 2012) and Subramanian (2015) and Majumder (2019).

Although, the composite index proposed by Majumder (2015) did not gain popularity till date, in past several years it drew attention of some authors like Subramanian (2015), Osberg (2017), and Josa and Aguado (2020). Whereas, Subramanian (2015) leaves a positive impression on the work considering its pro transfer-sensitive properties (obeying the principal of diminishing transfer), Osberg (2017) remained somewhat negative perceiving it as an extension of Gini coefficient with a postulation of its uneasy graphical representation as compared to that of the latter. The third study remains neutral with a mention of the mere existence of Majumder (2015) in literature as an extension of Gini coefficient.

As above, the primary objective of this paper is to rediscover the composite index proposed by Majumder (2015) to understand that: (i) it is a composite index – (a) when applied for each individual income group, it is equivalent to the refractive index of geometrical optics, which indicates extent of bending of the Lorenz curve from the egalitarian line, (b) when applied for the whole Lorenz curve framework, after addition of the values of the refractive index for all the income groups, it is equivalent to the Amato-Kakwani index, as some authors named it (see

Subramanian, 2015); (ii) it is not an extension of Gini coefficient; and (iii) its derivation is fully compatible with the visual appeal of the Lorenz curve framework and that of refraction of light.

The secondary objective of the paper is to remove all the pitfalls, though minor only, associated with the preliminary derivation and the ornamental aspect of the said index in Majumder (2015).

The paper is organised as follows. The second section is on the analogy of the graphical representation of refraction of light and the Lorenz curve. The third section discusses the concept of refractive index and the law that governs it in geometrical optics. The fourth section introduces the refractive inequality index for each stratum or income group under the Lorenz curve. The fifth section derives the overall index of inequality for the whole Lorenz curve framework. The sixth section is on the interpretation of the composite index. The seventh section examines the axioms and desirable properties of it. The eighth section poses some notes on the use of the measure. The ninth section presents some comparisons of it with the use of generalised entropy class of measures followed by conclusion and references.

2. The graphical representation of refraction of light and the Lorenz curve

[Please insert figure 1 about here]

[Please insert figure 2 about here]

The analogy of refraction of a ray of light and the deviation of Lorenz curve is shown with the help of the above figures 1 and 2 respectively. In figure 1, a ray of light refracts while passing from a transparent medium (say, air) into a dense medium (say, water). Figure 2 shows a Lorenz curve framework with five income groups or strata. It is well-understood that the fifth quintile, in the right-hand side, has the highest concentration of wealth or income. If one begins from the right-hand side top corner, one can observe that the Lorenz curve gets deviated each time while passing from one stratum into another. It is a simple matter of comprehension that, as light refracts according to the characteristics of different media, so also Lorenz curve does according to concentration of wealth or income in different strata. In the ideal case, when everyone has equal share of income or wealth, the Lorenz curve (or the ray of light) passes diagonally without refraction. As the property of deviation in a Lorenz curve framework is analogous to that of a fundamental principle of ray of light in physical science, the methodology associated with the latter is adopted in the former to derive a suitable measure of economic inequality in a scientific manner without any subjective assumptions or considerations. Angle of refraction or deviation of each segment of the Lorenz curve (such as θ_w, as shown in figures 1 and 2) matters in the process of derivation.

3. Refractive index and the law that governs it

In geometrical optics, propagation of light is governed by the Snell's law of refraction (see Jenkins and White 1981, pp. 9-13). It exhibits the relationship between different angles of a ray of light as it passes from one homogeneous transparent medium into another as follows:

$$r_a.sin(\theta_a) = r_w.sin(\theta_w), \tag{1}$$

where r_a is the refractive index of the medium 'a' the light is leaving, θ_a is the angle of incidence, r_w is the refractive index of the medium 'w' the light is entering, and θ_w is the angle of refraction. An illustration of refraction (from air to water) with usual notations is shown in figure 1.

In equation (1), we are interested to know r_w, where

$$r_w = \frac{\sin(45^0)}{\sin(\theta_w)}, \tag{2}$$

as $r_a = 1$ (refractive index of air is equal to one), $\theta_a = 45^0$.

In equation (2), the value of refractive index (r_w) can be determined when sin (θ_w) is known. We try to find the value of it in the context of Lorenz curve in the next section.

4. The refractive inequality index

When equation (2) is considered in the context of a Lorenz curve framework (as shown in figure 2), r_i may be termed as refractive inequality index (after replacing the suffix w by i, in general, for the i^{th} stratum or income group, where i = 1, 2, …, n). The value of sin (θ_i) for the stratum or income group (after replacing the suffix w by i, as above) is nothing but the perpendicular divided by the hypotenuse in each respective triangle, as shown by the dotted and solid lines in figure 2. As we deal with the angle at the right-hand side top corner, the perpendicular p = proportion of population in each equally sized group, which is nothing but 1/n, where n = number of strata or income groups, and the hypotenuse h_i = part of the Lorenz curve. In such a case, $\sin(\theta_i) = p/h_i$, where, $h_i = \sqrt{p^2 + y_i^2}$, where y_i = proportion of income in each group, such that $\sum y_i = 1$. If such values are put in equation (2), we get refractive inequality index of the form:

$$r_i = \frac{n}{\sqrt{2}}.h_i, \tag{3}$$

as, sin (45^0) = 1/√2. It says that refractive inequality index for a stratum or income group is nothing but the relevant part of the Lorenz curve multiplied by a constant n/√2. Further, as √2

= length of the Lorenz curve in ideal condition, refractive inequality index may also be expressed as the ratio of the part-length of the Lorenz curve in ideal condition to part-length of the Lorenz curve of the i^{th} stratum.

The working formula of the refractive inequality index (RII henceforth) may take the following forms:

$$r_i = \frac{n}{\sqrt{2}}.\sqrt{p^2 + y_i^2}. \quad \text{or} \tag{4}$$

$$r_i = \frac{1}{\sqrt{2}}.\sqrt{1 + (ny_i)^2}. \tag{5}$$

When, income share of a particular group $y_i = 0$, $r_{min} = 1/\sqrt{2} = 0.707$. In the ideal condition, when $p = y_i = 1/n$, $r_{ideal} = 1.00$. In the extreme case, when one person or group assumes all income (i.e., when $y_i = 1$) the maximum value of the index depends upon n, as can be confirmed from equation (5). For example, if there are five income groups (i.e., n = 5), $r_{max\,(5)} = 3.605$; for ten income groups (i.e., n = 10), $r_{max\,(10)} = 7.106$; when, n = 100, $r_{max\,(100)} = 70.714$; if n = 1000, $r_{max\,(1000)} = 707.107$ and so on. It appears that when n is large, $r_{max\,(n)} = n{*}r_{min}$. It is observed that, the extreme value of the index (when one individual or group assumes all income) increases with n. In words of Theil (1967, p. 92), it may seem objectionable that the upper limit of the index increases when the number of individual or group increases. Following the answer he provided in case of the entropy measure, we may also accept an index value of 7.106 when nine out of 10 persons (90 %) assume no income and an index value of 70.714 when 99 out of 100 persons (99%) assume nothing.

In order to gather empirical evidence, the World Income Inequality Database of 06 May 2020 (UNU-WIDER, 2020) is explored[1]. It is found that in case of quintile distributions with 6846 valid cases, the range of r_i is 0.707 to 2.963. In case of decile distributions of the same database with 6567 valid cases, r_i varies from a minimum value of 0.707 to the maximum value of 5.017.

5. The overall index of inequality for the whole Lorenz curve framework

When values of the RII for all the strata are added together, from equation (2) we may comprehend that in the right-hand side, we will have $\sum h_i$, which is nothing but the complete length of the Lorenz curve (as h_i represents part of that). In the left-hand side, we will have $\sum r_i$. Let $\sum h_i = M$ and $\sum r_i = X$, then from equation (2) we get:

$$X = \frac{n}{\sqrt{2}}M. \tag{6}$$

[1] Relevant SPSS command codes (for row-wise data) are presented in sections C and D of the Annexure – I.

We know that the length of the Lorenz curve varies between √2 and 2. It implies that value of X varies between n and n√2. After rescaling the above in '0 to 1'- point scale, we get:

$$R = \frac{X-n}{n\sqrt{2}-n}. \tag{7}$$

After a simple manipulation of the above [if we put the value of X from equation (6) to equation (7) and simplify both the denominator and numerator], the overall refractive inequality index (ORII henceforth) will take the following form:

$$R = \frac{M-\sqrt{2}}{2-\sqrt{2}}\,. \tag{8}$$

One may confirm that the above expression is nothing but the inequality measure based on the length of the Lorenz curve, as proposed by Amato (1968, p. 261), Lombardo (1969), Scala (1969) and Kakwani (1980a, pp. 83-85).

In order to suggest a working formula of the ORII, we may re-write M in equation (8) as follows:

$$R = \frac{(1/n\mu)\sum_{i=1}^{n}(\mu^2+y_i^2)^{\frac{1}{2}}-\sqrt{2}}{2-\sqrt{2}}\,, \tag{9}$$

where μ = average income share = 1/n. A complete derivation of M, as above, is available in Majumder (2019), which was derived to give it a similar shape as done by Kakwani (1980a, p. 84) for a continuous function:

$$L = \frac{1}{(2-\sqrt{2})}\left[\frac{1}{\mu}\int_0^{\infty}\sqrt{\mu^2+x^2}f(x)dx-\sqrt{2}\right], \tag{10}$$

where L = 'A new inequality measure' (as mentioned by him), which is based on the length of the Lorenz curve. Presentation of the above two formulae in their particular forms is purposive to define some basic properties of the index in the seventh section.

6. Interpretation of the composite index

Interpretation of the refractive inequality index (RII) and its values need some special mentions. As an index value of 1.00 represents the ideal condition, it is desirable for each of the strata (where proportion of population in a group is equal to income share of that group, i.e., $p = y_i$). Any deviation of the value of RII from 1.00 is undesirable. Any value of it from less than 1.00 is strictly undesirable (where proportion of population is greater than income share of that group, i.e., $p > y_i$). Standard literature in optics maintains that an index value of less than 1.00 does not represent a physically possible system (Nave, 2012). Further, in case of light, a refractive index value of less than 1.00 represents an 'anomalous refraction' (Feynman, 2011, p. 33-9). A phenomenon, which is anomalous in the field of physical science, has relevance in the field of economics of inequality too. Thanks to our latent feelings, we also realise that an income or wealth distribution with cases of $p > y_i$ (as mentioned above) is anomalous, which warrants redistribution of resources favouring the worse-off ones to correct the situation. An index value of more than 1.00 is also undesirable, as it indicates higher concentration of income or wealth in that group.

As above, one should read the RII and interpret its values in contrast to its value in the ideal condition, i.e., 1.00. For example, when it has a value of 0.71, the inequality condition of the respective income group or the stratum rests 0.71 points below the ideal condition; when it assumes a value of 1.42, the inequality condition of the income group or the stratum remains 0.42 points above the ideal condition. So, when index value increases from its minimum (0.71) to that of the ideal condition (1.00), it is good and desirable. Similarly, a fall in index value from its theoretical maximum (say, 3.61 for n = 5) towards that of the ideal condition (1.00) is good and desirable.

Also, one may imagine (in continuous case) that there is a point on the Lorenz curve where the slope of the tangent line is equal to that of the diagonal one. This may be called as the point of inflection, as it divides the population (and the Lorenz curve) into two groups (sections) with an RII value of less than 1.00 in the left and more than 1.00 in the right. So, an increase in value of RII up to 1.00 in the left, and a decrease in value of RII to 1.00 in the right indicating a redistribution of resources from the right segment of the Lorenz curve to the left are always desirable.

The overall refractive inequality index (ORII) varies from 0 to 1.00. While 0 means absence of inequality, 1.00 indicates maximum inequality. In the empirical exercises, it was presented after multiplying by 100.

7. Axioms and desirable properties of the RII & ORII

A good measure of economic inequality should have certain desirable properties (which are also known as axioms), from which it is possible to understand about how a measure behaves in responses to changes in its parameters. However, Kakwani (1980), Arnold (2012) and Subramanian (2015) very systematically proved most of the desirable properties of the index ORII (which is based on the length of the Lorenz curve) mathematically. In order to avoid replication of those mathematical derivations, this paper goes for numerical examples to understand the desirable properties of the said index. Following Bellù and Liberati (2006), primarily, five main axioms are considered: (i) the principle of transfers (also known as the Pigou-Dalton transfer principle), (ii) scale invariance, (iii) translation invariance, (iv) the principle of population replication, and (v) decomposability. Further, after testing the Pigou-Dalton principle of transfer, the workability of ORII is also tested in the context of some other essential properties, such as, the principle of pro transfer-sensitivity, as discussed by Subramanian (2015), and that of the principle of diminishing transfer, as discussed by Kolm (1976) and others.

In order to test whether the RII and ORII obey the axioms, this paper presents some numerical examples in tables 1 and 2 below[2]. Table 1 is self-explanatory. The first column shows individuals or income groups. The second column shows incomes (in any standard unit). The next five columns show transfer of incomes as per the first three principles under discussion. For example, the third, fourth and fifth columns are considered to test the principle of transfers. The third column shows that a transfer of 200 units of income takes place between the second poorest and the poorest groups. The fourth column shows the same amount of transfer between the richest and the second richest groups. The fifth column shows a transfer from the second richest group to the second poorest group. The sixth column shows an augmentation of income in each group by 20 per cent. The seventh column shows addition of income by 300 units in each group. The final column depicts a new income distribution (to test the fourth principle), which is an exact replication of the first one with ten individuals or groups instead of five.

Table 2 displays: (i) the values of the RII (according to individuals or groups, as in table 1), (ii) column-wise summation of index values, and (iii) and the ORII, when the values of the seventh column are standardised to put in '0 to 1'- point scale.

[Please insert table 1 about here]

[Please insert table 2 about here]

7.1. The Pigou-Dalton principle of transfers

One may consider that a rank-preserving progressive transfer of income takes place between a pair of individuals or groups. In such a situation, Pigou-Dalton transfer principle requires a fall in the index value and vice-versa. The second column of table 1 shows an initial distribution. The third column of it shows a transfer of 200 units of income from the second poorest group to the poorest one. It can be seen that after such a transfer, the values of both the Gini coefficient and the ORII decreased from 26.40 to 25.60 and from 6.99 to 6.50 respectively.

[2] Relevant SPSS command codes (for column-wise data) are presented in sections A and B of the Annexure-I. In section B, the formula for computing Gini coefficient was proposed by Majumder and Kusago (2020).

The numerical example shows that both the Gini coefficient and the ORII obey the Pigou-Dalton principle of transfer.

The same exercise is repeated again for the data in the fourth column, where a transfer of the same amount (200 units) took place between the richest and the second richest groups or individuals. After such a progressive transfer, the values of both the Gini coefficient and the ORII decreased from 26.40 to 25.60 and from 6.99 to 6.87 respectively. The numerical example re-confirms that both the Gini coefficient and the ORII obey the Pigou-Dalton principle of transfer.

As above, one may realise that in the third column of table 1, thanks to transfer of income, the poorest group is benefitted. On the contrary, in the fourth column, thanks to the same amount of transfer, the second richest group is benefitted. Although the contexts of two transfers are completely different, Gini coefficient (25.60) remains indifferent between the two transfers. So, the above two examples imply that Gini coefficient is transfer-neutral. On the contrary, ORII is pro transfer-sensitive meaning more sensitive to transfers at the lower levels of income (as the decrease in ORII is higher in the former than in the latter).

To put the matter technically, one may imagine that a given rank-preserving progressive transfer of income takes place between two pairs of individuals or groups such that the individuals or groups in each pair are separated by both a fixed number and a fixed income. In that case, following Subramanian (2015), we can say that an inequality measure is anti transfer-sensitive / transfer-neutral / pro transfer-sensitive, depending on whether the diminution in index value following the transfer between the poorer pair of individuals is lesser than / the same as / greater than the diminution in index value following the transfer between the richer pair of individuals.

As above, it may be postulated that an inequality measure (say, Z), which satisfies the Pigou-Dalton transfer axiom, will be ante transfer-sensitive if Z(b) > Z(c) > Z(d); transfer-neutral if Z(b) > Z(c) = Z(d); and pro transfer-sensitive if Z(b) > Z(d) > Z(c), where (c) and (d) are two different forms of the initial distribution (b), as shown in the second, third and fourth columns of table 1. In case of Gini coefficient, G(b) [= 26.40] > G(c) = G(d) [= 25.60]: the Gini coefficient is transfer-neutral. In case of the overall refractive inequality index, ORII (b) [= 6.99] > ORII (d) [= 6.87] > ORII (c) [= 6.50]: ORII is pro transfer-sensitive.

Theoretically, we know from a lemma (see Kakwani 1980a, p. 67) that any inequality measure that is equal to the arithmetic mean of a strictly convex function of income, satisfies Pigou-Dalton transfer axiom. Equation (9) expressing the ORII or equation (10) expressing the 'new inequality measure' of Kakwani (1980a, p. 83), shows that the index is the arithmetic mean of a strictly convex function of income, which implies that it is sensitive to transfers at all levels of income. In regard to transfer-sensitivity property, Kakwani (1980a, pp. 84-85) proved another lemma to show that the index (L) he proposed, attaches higher weight to transfers at the lower end than at the middle and upper ends of a distribution. The importance of such a weighting system has also been discussed by him in another occasion (see Kakwani, 1980b). However, according to him, unlike the (area-based) Gini coefficient, this measure (based on the curve length) is more sensitive to transfers at the lower levels of income, making it particularly applicable to problems such as measuring the intensity of poverty. As, ORII is equivalent to the 'new inequality measure' of Kakwani (1980, p. 84), properties of the latter are equally applicable for the former (i.e., for the ORII).

In particular, the point of giving more weight to transfers that decreases monotonically as income increases, has been discussed with great interest by Kolm (1976), Mehran (1976), Chateauneuf, Gajdos and Wilthien (2002), Rohde (2008) and others. In his seminal paper, Kolm (1976) discussed about the issue under the title of 'the principle of diminishing transfers'. He postulated that after having satisfied with the Pigou-Dalton principle of transfer, one may go a step further and value more such a transfer between persons with given income difference if these incomes are lower than if they are higher. Thus, he would prefer to transfer one pound from a person who earns 500 pounds a month to another one who earns only 100, than to transfer one pound from a 900 pounds earner to a person who already earns 500 pounds. In line with the same thought, if we were asked that of the two transfers (as cited above in the context of table 1), which one do we prefer – obviously we will favour (c) over (d), as comparatively poorer people are benefitted in that than in the latter. With this level of mindset and priority, one may obviously find it difficult to continue with Gini coefficient in all occasions, as it remains transfer-neutral. On the good side, the ORII, which is equivalent to the inequality measures based on the length of the Lorenz curve, remains in advantageous position with its built-in weighting system on transfers favouring the worse-off ones.

Table 2 shows values of the refractive inequality index (RII) corresponding to six income levels in each of the second through seventh columns of table 1. We know from section 6 that when value of $r_i < 1.00$ under the left segment of the Lorenz curve, it is strictly undesirable. An increase in value of r_i up to 1.00 is desirable. It is seen that after the first transfer from the second poorest group to the poorest group, r_1 increased from 0.75 to 0.78. At the same time, r_2 also decreased from 0.84 to 0.81. As the initial value of $r_2 < 1.00$, a decrease in income share results further decrease in the value of r_2, which is not desirable. However, as the poorest group is benefitted and as ORII attaches more weight to transfer at the lower levels of income, ORII decreased from 6.99 to 6.50.

When transfer of income takes place from the richest group to the second richest group (as in the fourth column of table 1), r_5 decreases from 1.36 to 1.30 in table 2, which is desirable. At the same time, for increase in income share in the fourth group, r_4 increases from 1.19 to 1. 25. As the initial value of $r_4 > 1.00$, a further increase in that is not desirable. However, as a result of overall change, ORII decreased from 6.99 to 6.87.

We know that the point of inflection (as discussed in section 6) divides the population (and the Lorenz curve) into two groups (sections) with an RII value of less than 1.00 in the left and more than 1.00 in the right. When transfer of income takes place from one such group to another, RII also maintains the spirit of the Pigou-Dalton transfer axiom, as long as $r_i \leq 1.00$ in the left and $r_i \geq 1.00$ in the right respectively. The fifth column of table 1 shows such a transfer and one may confirm from the fifth column of table 2 that after such a transfer, r_4 decreased from 1.25 to 1.13 indicating a decrease in inequality under the right section of the Lorenz curve. At the same time, under the left section of the Lorenz curve too, r_2 increased from 0.84 to 0.88 indicating a decrease in inequality. As a result of the overall decrease in inequality conditions under the both segments, the ORII also decreased from 6.99 to 6.24.

7.2. The principle of scale invariance

The property of scale invariance requires the inequality measure to be invariant to equi-proportional changes of the original incomes. For example, from the sixth column of table 1, one may understand that the original incomes (as in the second column of table 1) are multiplied by 1.2, to observe a 20% increase for each group / individual. After such a change, one may see that inequality measures in tables 1 and 2 (the sixth column in both) remain the same. This proves that ORII and its components satisfy the property of scale invariance.

7.3. The principle of translation invariance

The property of translation invariance requires the inequality measure to be invariant to uniform additions or subtractions to original incomes. One may see that in the seventh column of table 1, income for each group is augmented by 300 units as compared to the original incomes displayed in the second column of the same table. Results show that ORII and its components do not satisfy the property of translation invariance. One may realise that after the augmentation of income by a fixed unit, income shares under the left section of the Lorenz curve increased and the same under the right section of the Lorenz curve decreased. As a result, Gini coefficient and ORII also decreased.

7.4. The axiom of the principle of population

The axiom of the principle of population requires the inequality measure to be invariant to replications of the original population. The final column of table 1 shows such a replication and results show that both the Gini coefficient[3] and ORII satisfy the axiom of the principle of population.

7.5. The axiom of decomposability

An index of inequality may be said to be additively decomposable if for any grouping total inequality can be written as the sum of: (i) a between-group component, and (ii) a within-group component. This property allows the unambiguous measurement of the contribution of a particular grouping (or variable) to overall inequality (Anand, 1983, p. 87, 319). With some level of critical reasoning, one may realise that if grouping is not done before testing the axiom, the point of considering the second component of 'within-group' inequality will become void. As grouping of observations causes some amount of shortfall in a summary measure (say, in Gini coefficient) as compared to that computed from micro-data (see Majumder and Kusago, 2018) because of ignoring the within group inequality, the 'within-group component' is considered under the axiom. If one computes a summary measure (say, Gini coefficient) for n = 10, and goes for testing the axiom of decomposability without any grouping, she/he needs to consider the between-group component only; the question of within-group component does not arise. One may realise that the component of within-group inequality loses its point, when the axiom of decomposability is considered in the context of the ORII too. ORII is obtained after adding inequality condition of each individual or group under a study. So, ORII is additively decomposable, by definition. The matter can be presented in the following way.

$$ORII \equiv \sum r_i = r_1 + r_2 + \cdots + r_n. \tag{11}$$

It is shown, as above, that ORII (or equivalently the Amato-Kakwani inequality index, which is based on the length of the Lorenz curve) satisfies most of the desirable properties to be a good measure of economic inequality. Most importantly, it satisfies the principles of pro-transfer-sensitivity (or that of the diminishing transfer) and decomposition. It was mentioned by Chateauneuf, Gajdos and Wilthien (2002) that the Atkinson, Kolm and Theil indices respect the principle of diminishing transfer. According to Rohde (2008), the introduction of the diminishing transfer property and decomposition principle has reduced the range of viable indices to a subset of the Generalised Entropy class of measures. However, the ORII and

[3] When the formula of Gini coefficient is obtained for small n following the 'principle of mean difference without repetition' as proposed by Kendall (1948, p. 42), or simply when it is corrected for small n replacing n^2 in the denominator by n*(n-1), it does not satisfy this axiom.

equivalently the Amato-Kakwani inequality index will also come under this category, as they satisfy both the above-mentioned crucial properties.

8. Notes on the use of RII and ORII

8.1. The composite nature of the index

Thanks to the detailed derivation and discussions, it is now clear that the index under consideration is a composite one – it can be applied in parts and as a whole.

When applied in parts, one may obviously raise question that while the income distribution table is available, what is the point of using refractive inequality index (RII) for each individual or income group or stratum? Authors like Piketty (2014, p. 266) suggested using of income shares from distribution tables to evaluate inequality conditions of individuals or groups. In order to supplement the result of a summary measure, Osberg (2017) suggested to examine visually the relevant section of the Lorenz curve. However, one may realise that use of an income share either from a distribution table or from a Lorenz curve to understand inequality condition, does not complement the use of RII for the same purpose. Summation of income shares (say, for a quintile distribution) does not lead to an inequality measure (as it is always equal to one), although the same (summation) in case of RII leads to a measure of economic inequality (such as, ORII or Amato-Kakwani inequality index). Also, a simple visualisation of figures from income distribution table or from Lorenz curve from normative perspective may be misleading if not quantified (with weights) in a proper manner.

Thanks to curiosity, the relationships between income share and refractive inequality index for quintile distributions are checked using data from the World Income Inequality Database of 06 May 2020 (UNU-WIDER, 2020)[4] and presented below.

[Please insert figure 3 about here]

[Please insert figure 4 about here]

One may examine that in figure 3, the relationship between the refractive inequality index (r_1) and income share (y_1) of the first group is quadratic. It implies that conversion rate of income share into index value is not constant throughout. It is lower at lower levels of income share and it gradually increases with the increase in income share. So, use of income share to explain inequality condition conveys different meaning than that of using the refractive

[4] Please see footnote 1.

inequality index for the same purpose. One may continue to check the said relationships (from figure 4 to figure 6) and may confirm that this phenomenon holds to be true up to the fourth income group. In case of the final income group of the quintile distribution, the relationship between r_5 and y_5 is linear, as shown in figure 7. It implies that the conversion rate of income share into index value is constant. In case of a quintile distribution, meaning of using the final income share or the corresponding RII value to read the inequality condition, is the same.

[Please insert figure 5 about here]

[Please insert figure 6 about here]

[Please insert figure 7 about here]

The same exercise was repeated for the decile income or consumption distributions available in the World Income Inequality Database of 06 May 2020 (UNU-WIDER, 2020)[5] and found similar results (not presented in the paper). To be more specific, r_i and y_i are quadratically related with an adjusted R-square value of 1.00 for each decile group for all i = 1, 2, …, 9. For the 10th income group, the relationship is perfectly linear with an adjusted R-square value of 1.00.

It appears from the above that use of refractive inequality index for first (n-1) groups for quintile and decile distributions does not complement the use of simple income shares to explain inequality conditions. Uses of the nth income share and the refractive inequality index for the nth group convey the same meaning. One may realise that uses of RII and ORII, instead of simple use of individual income shares, are more appropriate to satisfy the propositions put forward by Piketty (2014, p. 266) and Osberg (2017) on the subject matter.

8.2. RII and ORII are not extensions of the Gini coefficient

Neither the refractive inequality index (RII) nor the overall refractive inequality index (ORII) is an extension of the Gini coefficient. RII is an angle-based measure, which was derived in analogue to the refractive index of geometrical optics. Angle of deviation of a particular segment of the Lorenz curve with respect to the egalitarian line is the key issue that matters in derivation (instead of area covered by the egalitarian line and the Lorenz curve). Moreover, RIIs have no equivalent counterparts in Gini coefficient, as the latter is not additively decomposable (see Anand, 1983, p. 87). When RIIs are added and standardised, it becomes

[5] *Ibid*.

equivalent to the inequality measures based on the length of the Lorenz curve. Although not widely popular, the inequality measures based on the length of the Lorenz curve has distinctive place in literature. Authors who proposed it or discussed about it (as cited in section 5), have never considered it as an extension of Gini coefficient.

In Majumder (2015), some empirical exercises were presented to show that the Gini coefficient and ORII are perfectly correlated by quadratic equation with adjusted R-square value of 1.00. However, the intension was not to mean that the latter is an extension of the former. Mathematically, when in quadratic relationship, rate of change in one variable with respect to the other is not constant throughout – meaning that workability of each is different. In such a situation, the use of one does not perfectly substitute the use of the other.

An ornamental dimension was also added to Majumder (2015), where refractive index values of precious gem stones were compared with the same of different income groups respectively in order of hierarchy (in descending order). Surprisingly, it was found that the refractive inequality index of the richest group of a highly unequal quintile distribution is closer to that of a piece of a diamond (2.42). The paper tried to compare the same of other income groups too (in order of magnitude of the index value) with the refractive index of other precious gem stones in order of their hierarchy. Although, such a presentation was too attractive to some readers, others raised question that whether one needs to understand gemmology to read the work of Majumder (2015)[6]. The simple and brief answer is 'no'. One may ignore the ornamental dimension of Majumder (2015) summarily. It will have no impact to study inequality conditions purely under the Lorenz curve framework.

8.3. Visual appeal of RII and ORII

Visual appeal of the Gini coefficient is solely related to that of the Lorenz curve. The same of the RII and ORII is related to that of the Lorenz curve and additionally to that of the refraction of light, as illustrated in figure 1. Their approach of derivation is fully compatible not only with the visual appeal of the Lorenz curve framework in reality, but also one may go beyond it with some fantasy considering the unit-square of the Lorenz curve framework as a World under the Sun, where the egalitarian line is nothing but a ray of light that touches everyone uniformly without any discrimination or refraction. When uniformity breaks, the ray of light refracts as Lorenz curve does in reality. So, the graphical illustrations of the RII and ORII are more optically appealing and their academic spirit is in no way compromised.

[6] Most of such comments received during oral presentation in different occasions and by emails.

9. A comparison with some Generalised Entropy class of measures

As a background exercise of this paper, the workability of the following indices is tested[7] as per data available in the first four columns of table 1. First, Atkinson inequality index was considered (see Atkinson, 1970). It works with an inequality aversion factor (say ε), where ε varies from 0 to ∞. One may choose appropriate value of ε to make it pro transfer-sensitive. According to this measure, higher values of ε indicate more weight to transfers at the lower end of a distribution and (simultaneously) less weight to transfers at the upper end (Atkinson 1970). The recommended and the commonest used values of ε are: 0.5, 1, and 2 (Anand 1983, pp. 84-85). In the present exercise, the values of ε considered are: 0.25, 0.5, 1, 2 and 10. Index values are displayed in table 3. All the first four variants of Atkinson index are pro-transfer sensitive. One may verify that visibly for all $\varepsilon \leq 6$, measures are pro transfer-sensitive. As ε increases thereafter, the weight given at the upper end virtually becomes nil. At least visibly, Atkinson index with $\varepsilon = 10$ seems to be the classical example of Rawlsian function mini {yi} as $\varepsilon \rightarrow \infty$, where interest of the poor only is considered ignoring completely the transfers among rich (Atkinson 1970; Anand 1983, p. 83). So, when we say that Atkinson index satisfy the principle of diminishing transfer, we need to understand that practically, we may go up to $\varepsilon = 5$. However, when $\varepsilon \geq 1$, Atkinson index becomes undefined for zero share of income in one group. In order to avoid such complicacies, one needs to consider a variant of Atkinson index with $0 < \varepsilon < 1$. It is found after doing an empirical exercise using the decile dataset (with 6567 observations) from the World Income Inequality Database of 06 May 2020 (UNU-WIDER, 2020)[8] that the Atkinson index with $\varepsilon = 0.25$ maintains a quadratic relationship with the ORII (and hence with the Amato-Kakwani inequality index) with an adjusted R-square value of 1.00.

Secondly, Theil's entropy index T (see Theil, 1967, pp. 91-95) and Theil's second measure L (see 1967, pp. 125-127), and in particular the Generalised Entropy Measure, as proposed by Shorrocks (1980) with the inequality aversion factor $\alpha \geq 2$ were considered. The inequality aversion factor of it (say α), varies from $-\infty$ to $+\infty$. It is to be remembered that $\alpha = 0$ and $\alpha = 1$ correspond to Theil's second measure L and Theil's entropy index T respectively. The commonest values used for this inequality aversion factor are: -1, 0, 1 and 2 respectively. In the present exercise, three values of α are considered: 0, 1 and 2. Results are presented in table

[7] Relevant SPSS command codes (for column-wise data) are presented in sections E, F and G of the Annexure-I.
[8] Relevant SPSS command codes (for row-wise data) are presented in section H of the Annexure-I.

3. It can be seen that Theil's L and Theil's T are pro transfer sensitive. The variant with $\alpha = 2$ is transfer-neutral. The variants with $\alpha > 2$ are ante transfer-sensitive, as discussed under the sub-section 7.1. It is to be noted that the Theil's L (with other variants of Generalised Entropy Measures with $\alpha < 0$) is undefined for zero share of income in a group. So, one may go with the Theil's T, which is pro transfer-sensitive and additively decomposable in weak sense of the term (see Anand, 1983, p. 309). It also maintains a quadratic relationship with ORII (and hence with the Amato-Kakwani inequality index) with an adjusted R-square value of 0.998, as tested with the same UNU-WIDER data[9].

Thirdly, extended Gini coefficient is considered (see Yitzhaki and Schechtman, 2005). Extended Gini coefficient works with an inequality aversion factor v, which varies from 0 to ∞. When $v = 2$, it becomes equivalent to the Gini coefficient. The weighting scheme of the index is similar to that of Atkinson index. The present exercise considers four values of v: 2.5, 3, 4, and 10. Results are displayed in table 3, which also follow similar pattern as they do in the case of Atkinson index. The first three are pro transfer-sensitive. Extended Gini coefficient with $v = 10$, conveys similar meaning as the Atkinson index with $\varepsilon = 10$ does. After repeating similar empirical exercises (as above)[10], it is found that Extended Gini coefficient maintains a power relationship with ORII (and hence with the Amato-Kakwani inequality index) with an adjusted R-square value of 0.997.

As above, those who prefer to work with ORII (or equivalently Amato-Kakwani inequality index) may also consider working with Atkinson index with $\varepsilon = 0.25$, and/or Theil's T, and/or Extended Gini coefficient with $v = 2.5$. If, however, someone: (i) tries to avoid arbitrary selection of weights to transfer, (ii) wants to follow the principle of pro transfer-sensitivity (or that of diminishing transfer), and decomposability in strict sense of the term, she/he may go with the ORII.

[Please insert table 3 about here]

10. Conclusion

Refractive inequality index, which was derived in analogue to the index of refraction of geometrical optics and the overall refractive inequality index, which is equivalent to the inequality measures based on the length of the Lorenz curve, may be used to study inequality

[9] *Ibid.*
[10] *Ibid.*

conditions along different segments of the Lorenz curve as well as for the whole framework respectively.

Declarations

Funding: The author did not receive support from any organisation for the submitted work.

Conflicts of interest/Competing interests: (i) The author has no relevant financial or non-financial interests to disclose. (ii) This article does not contain any studies with human participants or animals performed by the author.

Availability of data and material: The datasets generated during and/or analysed during the current study are available in the UNU-WIDER repository, https://www.wider.unu.edu/project/wiid-%E2%80%93-world-income-inequality-database.

Code availability: Relevant SPSS (15.0) command codes are presented in Annexure – 1.

Acknowledgement: The author is thankful to all for comments and suggestions.
Responsibility of any error rests with the author.

References

Amato, V., *Metodologia Staistica Strutturale vol.1*, Cacucci, Bari, 1968.

Anand, S., *Inequality and Poverty in Malaysia: Measurement and Decomposition*, Oxford University Press, New York, 1983.

Arnold, B.C., “Inequality Measures for Multivariate Distributions,” *METRON - International Journal of Statistics*, 63 (3), 317-327, 2005.

--------------., “On the Amato inequality index,” *Statistics and Probability Letters*, 82 (8), 1504-1506, 2012.

Atkinson, A.B., “On the measurement of inequality,” *Journal of Economic Theory*, 2 (3), 244-263, 1970.

Bellù, L. G. and Liberati, P., *Inequality and Axioms for its Measurement*, Food and Agriculture Organisation of the United Nations, Rome, 2006 (Retrieved August, 11, 2019, from http://www.fao.org/policy-support/tools-and-publications/resources-details/en/c/848434/).

Chateauneuf, A. and Gajdos, T., & Wilthien, P-H., “The Principle of Strong Diminishing Transfer,” *Journal of Economic Theory*, 103 (2), 311-333, 2002.

Feynman, R. P., *Feynman Lectures on Physics 1: Mainly Mechanics, Radiation, and Heat*, Basic Books, New York, 2011.

Jenkins, A. F. and White, H. E., *Fundamentals of Optics*, McGrawhill, New Delhi, 1981.

Josa, I. and Aguado, A., “Measuring Unidimensional Inequality: Practical Framework for the Choice of an Appropriate Measure,” *Social Indicators Research*, 149, 541-570, 2020.

Kakwani, N., *Income Inequality and Poverty: Methods of Estimation and Policy Applications*, Oxford University Press, New York, 1980a.

----------------., “On a Class of Poverty Measures,” *Econometrica*, 48 (2), 437-446, 1980b.

Kendall, M. G., *The Advanced Theory of Statistics, Vol. 1, (Fourth edition),* Charles Griffin, London, 1948.

Kolm, Serge-Christophe., “Unequal inequalities. I,” Journal of Economic Theory, 12 (3), 416-442, 1976.

Lombardo, E., “Nota Sulla Concentrazione Secondo GINI-LORENZ,” *Quaderni dell’Istituto di Statistica, Università degli Studi di Roma, Facolt`a di Economia e Commercio, Roma*, 5, 137-142, 1969.

Majumder, A., “An alternative measure of economic inequality in the light of optics,” *ECINEQ Working Paper 2014-346*, 2014 (Retrieved August 11, 2019, from http://www.ecineq.org/milano/WP/ECINEQ2014-346.pdf).

------------------., “An alternative measure of economic inequality under the Lorenz curve framework in analogue to the index of refraction of geometrical optics,” *Economics Bulletin*, 35 (2), 1076-1086, 2015.

Majumder, A. and Kusago, T., “A note on the use of decile or quintile group-share of income or consumption from the popular income inequality databases to explain inequality conditions,” Economics Bulletin, 38 (4): 2152-2166, 2018.

------------------., “An Appraisal of the Pro Transfer-sensitive Measures of Economic Inequality with Left-leaning Considerations,” *Artha Vijnana (Journal of Gokhale Institute of Politics and Economics)*, 61 (2), 119-142, 2019.

Majumder, A. and Kusago, T., “A Simple Working Formula of Gini Coefficient with Some Common Software Command Codes,” *Artha Vijnana (Journal of The Gokhale Institute of Politics and Economics)*, 62 (3), 219-224, 2020.

Mehran, F., “Linear Measures of Income Inequality,” *Econometrica*. 44 (4), 805-809, 1976.

Nave, R. C., “Refraction of Light. HyperPhysics,” Atlanta, 2012 (Retrieved August 11, 2019, from http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/refr.html).

Osberg, L., “On the Limitations of Some Current Usages of the Gini Index,” *Review of Income and Wealth*, 63(3), 574–584, 2017 (doi:10.1111/roiw.1225619-142).

Piketty, T., *Capital in the Twenty-First Century,* Harvard University Press, Cambridge, 2014.

Rohde, N., “A Comparison of Inequality Measurement Techniques,” *Issue 377 of Discussion paper, PANDORA electronic collection*, University of Queensland, School of Economics, 2008.

Scala, C., “Nota sulla sensibilità di un indice di concentrazione,” *Atti della XXVI Riunione Scientifica della Società Italiana di Statistica, Firenze*, 2: 425-433, 1969.

Shorrocks, A. F., “The Class of Additively Decomposable Inequality Measures,” *Econometrica*, 48 (3), 613-625, 1980.

Subramanian, S., “More tricks with the Lorenz curve, *Economics Bulletin*,” 35 (1), 580-589, 2015.

Theil, H., *Economics and Information Theory*, North Holland, Amsterdam, 1967.

UNU-WIDER., *World Income Inequality Database (WIID), 06 May 2020*, UNU-WIDER, Luxembourg, 2020.

Yitzhaki , S., & Schechtman, E., “The properties of the extended Gini measures of variability and inequality,” *METRON - International Journal of Statistics*, 43 (3), 401-433, 2005.

<u>(Figures)</u>

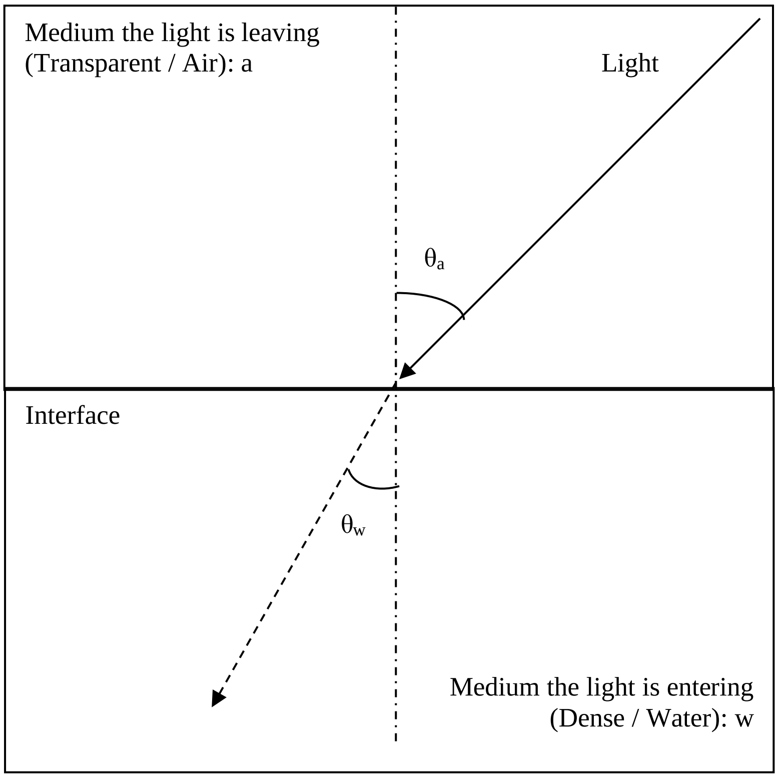

Figure 1. An illustration of refraction of light (with vertical normal)

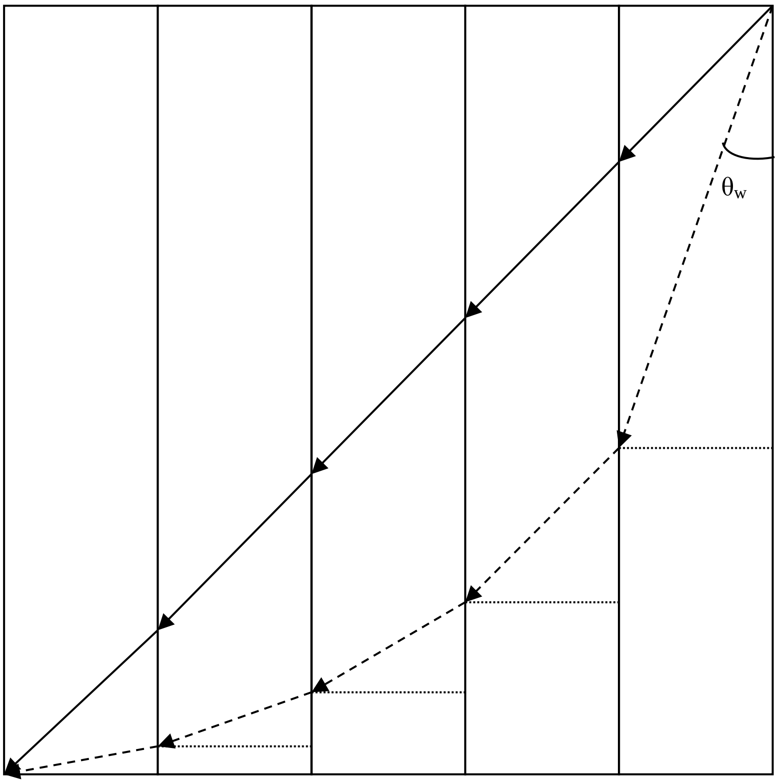

Figure 2. An illustration of Lorenz curve framework with five groups

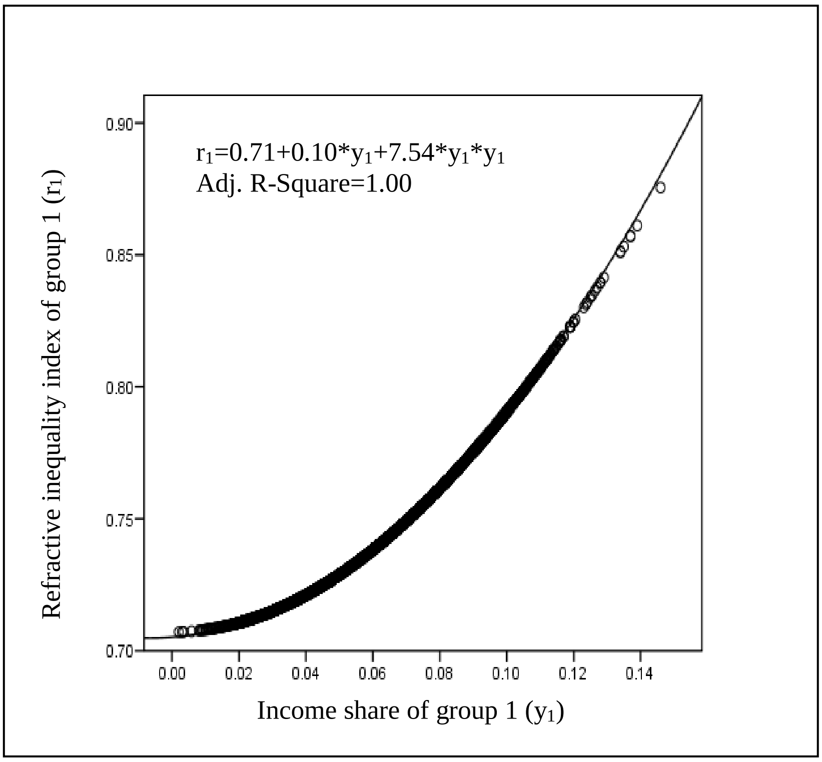

Figure 3. Relationship between r_1 & y_1 using the WIID of 6 May 2020; n = 6846

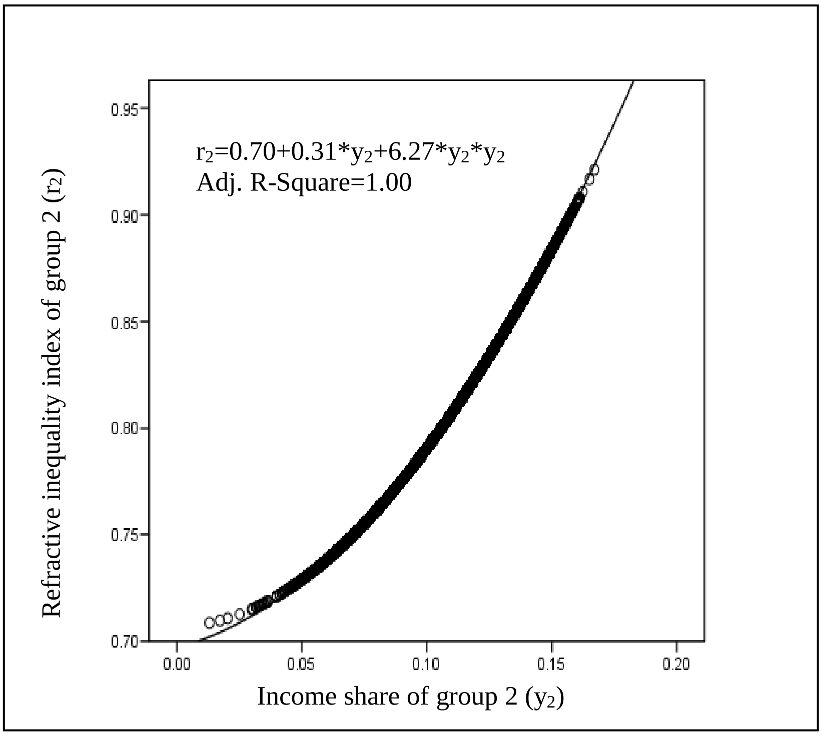

Figure 4. Relationship between r_2 & y_2 using the WIID of 6 May 2020; n = 6846

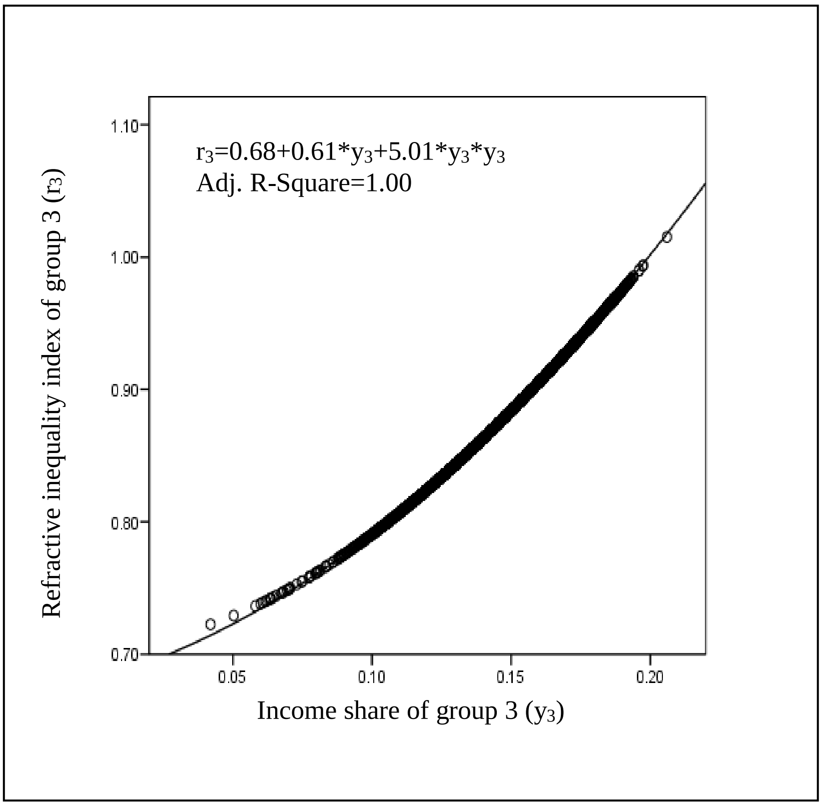

Figure 5. Relationship between r_3 & y_3 using the WIID of 6 May 2020; n = 6846

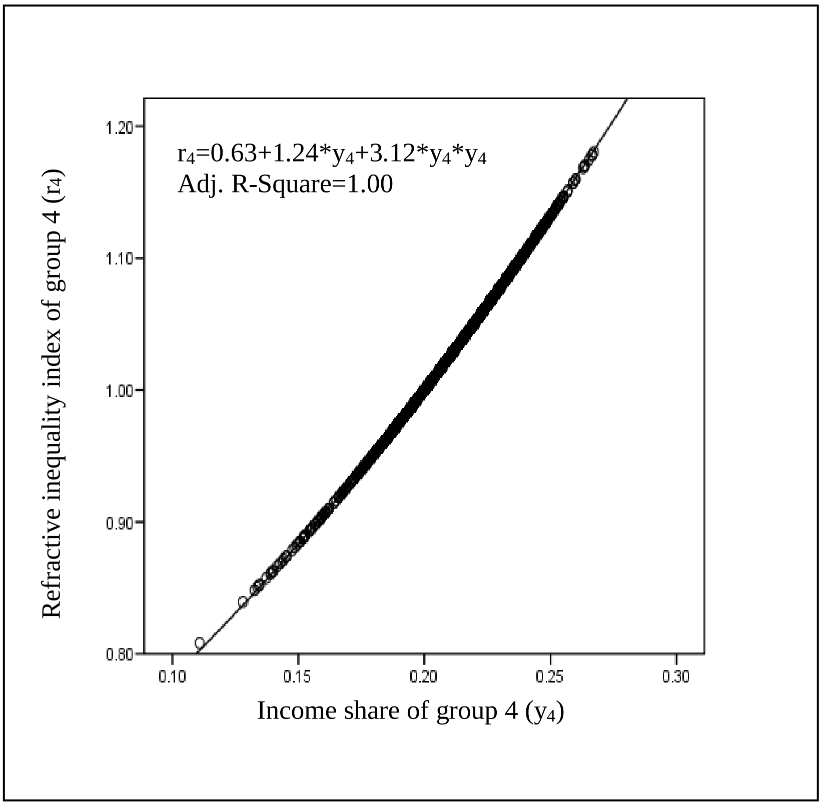

Figure 6. Relationship between r_4 & y_4 using the WIID of 6 May 2020; n = 6846

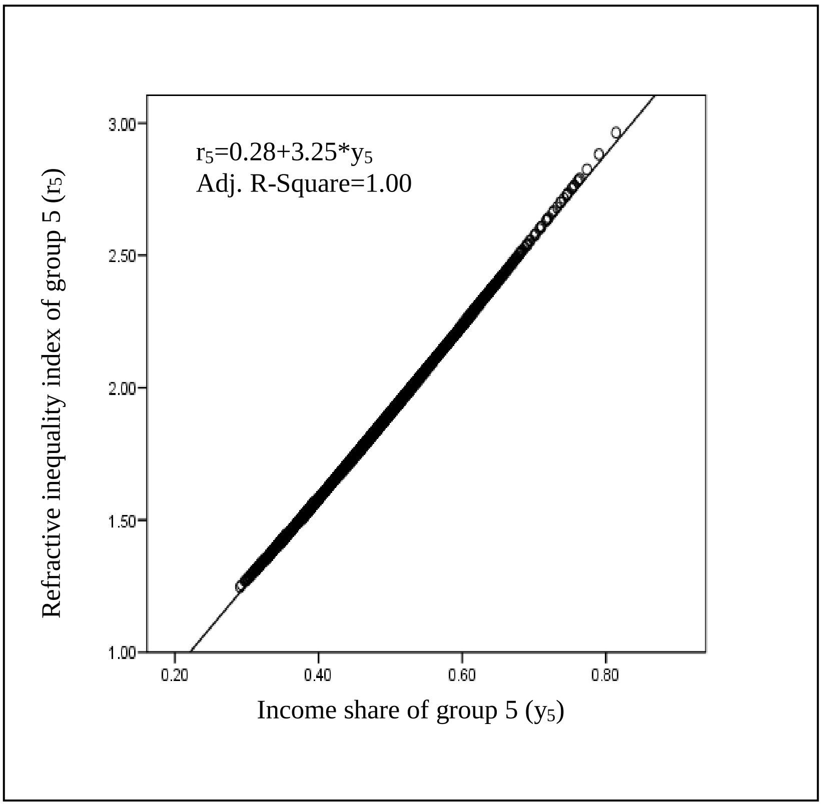

Figure 7. Relationship between r_5 & y_5 using the WIID of 6 May 2020; n = 6846

<u>Tables</u>

Table 1. Numerical examples to test properties of the Overall Refractive Inequality Index (ORII)

Individuals or Groups	Incomes	Principle of transfers			Sale invariance	Translation invariance	Principle of population
	(b)	(c)	(d)	(e)	(f)	(g)	
							700
							700
							1300
							1300
							2000
1	700	900	700	700	840	1000	2000
2	1300	1100	1300	1500	1560	1600	2700
3	2000	2000	2000	2000	2400	2300	2700
4	2700	2700	2900	2500	3240	3000	3300
5	3300	3300	3100	3300	3960	3600	3300
Mean income	2000	2000	2000	2000	2400	2300	2000
Total income	10000	10000	10000	10000	12000	11500	20000
G*	26.40	25.60	25.60	24.80	26.40	22.96	26.40
ORII*	6.99	6.50	6.87	6.24	6.99	5.21	6.99

*After multiplying by 100; G: Gini coefficient; ORII: Overall Refractive Inequality Index

Table 2. The components of the Overall Refractive Inequality Index

Distribution /Index	(b)	(c)	(d)	(e)	(f)	(g)
r_1	0.75	0.78	0.75	0.75	0.75	0.77
r_2	0.84	0.81	0.84	0.88	0.84	0.86
r_3	1.00	1.00	1.00	1.00	1.00	1.00
r_4	1.19	1.19	1.25	1.13	1.19	1.16
r_5	1.36	1.36	1.30	1.36	1.36	1.31
$\sum r_i$	5.15	5.14	5.14	5.13	5.15	5.11
ORII	6.99	6.50	6.87	6.24	6.99	5.21

r_i: Refractive inequality index (RII), * After multiplying by 100

Table 3. A comparison with the generalised class of measures (using the first four income distributions of table 1)

Index/Distribution	Atkinson index					Generalised Entropy Measure			Extended Gini coefficient			
	ε = 0.25	ε = 0.50	ε = 1	ε = 2	ε = 10	Theil's L	Theil's T	α = 2	v = 2.5	v = 3	v = 4	v = 10
(b)	3.00	6.13	12.71	25.84	58.16	13.59	11.68	10.90	23.86	21.12	16.22	3.63
(c)	2.75	5.57	11.23	21.72	47.09	11.91	10.87	10.50	22.86	20.00	15.04	3.13
(d)	2.94	6.03	12.55	25.71	58.16	13.42	11.41	10.50	23.5	20.96	16.19	3.63
(e)	2.68	5.51	11.55	24.20	58.15	12.27	10.44	9.70	22.36	19.84	15.39	3.59