A similar version of the paper appeared in Artha Vijnana

Artha Vijnana
Journal of Gokhale Institute of Politics and Economics
2019, 61 (2): 119-142. (ISSN: 0971586X)

An Appraisal of the Pro Transfer-sensitive Measures of Economic Inequality with Left-leaning Considerations

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An inequality measure is said to be pro transfer-sensitive when the diminution in index value following transfer between the poorer pair of individuals is greater than the diminution in index value following transfer between the richer pair of individuals. This property conforms to the prime but latent objective of studying economic inequality, which prioritises the claim of the worse-off of the two groups. However, such measures are either much less frequently employed or lesser known. This paper makes an appraisal of some of them synchronising various approaches and perspectives with an overt objective of making them popular.

I Introduction

Inequality measures under the Lorenz curve framework, considering the transfer sensitivity property, may broadly be classified into three categories: (i) anti transfer-sensitive, (ii) transfer-neutral, and (iii) pro transfer-sensitive. We may imagine that a given rank-preserving progressive transfer of income takes place between two pairs of individuals or groups such that the individuals or groups in each pair are separated by both a fixed number and a fixed income. In that case, following Subramanian (2015), we can say that an inequality measure is anti transfer-sensitive / transfer-neutral / pro transfer-sensitive, depending on whether the diminution in index value following the transfer between the poorer pair of individuals is lesser than / the same as / greater than the diminution in index value following the transfer between the richer pair of individuals. It follows that while the underlying weighting schemes of the first and the third are to favour the richer and poorer sections towards the right and left wings of the Lorenz curve respectively, that of the middle is balanced treating everybody equally. It is well understood that the third group only conforms to the left-leaning considerations and sounds in accordance with the prime but latent objective of studying economic inequality, which prioritises the claim of the worse-off of the two groups after any transfer. However, measures under this category are either much less frequently employed or less known. On the contrary, application of the transfer-neutral Gini coefficient is immensely vast irrespective of the nature of income distributions. Its application is inappropriate for distributions, which bulge at the bottom with higher concentration of wealth or income at the top representing left-leaning Lorenz curves. The ante transfer-sensitive measures (which favour the rich) are not suitable for common use except in some very special cases, where the interests of the well-off are to be protected.

In order to combat the issues associated with the transfer-neutrality of Gini coefficient, a number of generalised measures are evolved in literature which can be tuned to the extent of pro transfer-sensitive ones, thanks to their differing aversions to inequality. However, their use is also limited as compared with that of the Gini coefficient. In such a situation, the present study makes an appraisal of some pro transfer-sensitive measures in the Lorenz curve framework and demonstrates their workability (with simple numerical examples) in comparison with the Gini coefficient and some generalised measures with an overt objective of making them popular. We consider those pro transfer-sensitive measures only the visual appeal of which does not differ much from that of the Gini coefficient and its legacy. However, the emphasis of the paper is not on theoretical exaggeration but on the practical concerns of exposition, and on the advancement of easily comprehended and readily usable measures of inequality with desired properties.

Three pro transfer-sensitive measures considered are: (i) the length-based inequality measure, known as, 'Amato-Kakwani inequality index', as proposed by Amato (1968, p. 261) and independently by Kakwani (1980, pp. 83-85); (ii) the area-based left-wing Gini coefficient, as derived by Subramanian (2015); and (iii) the angle-based pair of inequality measures in analogue to the refractive index of geometrical optics, as proposed by Majumder (2015). In regard to the second, I would rather prefer it to be called as Subramanian's L index henceforth. In addition to these measures, three generalised inequality indices are considered for a comparison of their workability with the pro transfer-sensitive ones such as (i) Generalised entropy measure, (ii) Atkinson index of inequality, and (iii) Extended Gini coefficient.

Some empirical exercises (including some scatterplots at the appendix) are done using data from WIID 3.4 (UNU-WIDER 2017), which contain valid information of 4978 cases on decile group shares of income for 177 countries.

The rest of the paper is structured as follows. Section II discusses the workability of the Gini coefficient and its property of transfer-neutrality. Section III presents three pro transfer-sensitive measures of economic inequality, their properties and applications with numerical examples in six sub-sections. Section IV briefly discusses the workability of three generalised inequality measures. Section V has a comprehensive table showing characteristics of all the discussed measures. Section VI presents conclusion followed by references.

II Gini Coefficient and Its Transfer-neutrality

The Lorenz curve measures the extent to which a distribution of income or consumption expenditure among individuals or groups within an economy deviates from a perfectly equal distribution. In a unit square Lorenz curve framework, the perfectly equal distribution (or the ideal condition) is denoted by the 45^0 diagonal line, which is also known as the egalitarian line. The most popular measure of economic inequality, the Gini coefficient, measures the extent of deviation of income distribution considering the area covered by the Lorenz curve and the egalitarian line. If the said area is denoted by A, Gini coefficient = 2A. In addition to the main working formula under the Gini's mean difference approach (see Kendall 1948, pp. 42-45), there are more than a dozen of those to spell this measure (see Yitzhaki and Schechtman 2013, pp. 11-31).

Although, Gini coefficient satisfies Pigou-Dalton transfer axiom[1], it is transfer-neutral, i.e., it is not differentially sensitive to transfers at either the lower or upper end of an income distribution. This transfer-neutrality may seriously contradict with the importance of linking between changes in different segments of an income distribution and changes in an inequality measure that tries to summarise the distribution. To cite one numerical example, consider an income distribution: p = (7, 13, 20, 27, 33). We make two downward transfers separately in p (at the lower and upper ends respectively) to have two different forms of the distribution respectively: q = (9, 11, 20, 27, 33) and r = (7, 13, 20, 29, 31). It can be checked that Gini coefficients for the three are: 0.3300, 0.3200, and 0.3200 respectively. Pigou-Dalton transfer axiom is satisfied in both the cases, as after both the rank-preserving transfers, index values decreased. However, the Gini coefficient for both the altered distributions remains the same, although they are derived with completely two different objectives (in one, poorer section is benefitted while the richer in the other). It appears that Gini coefficient may unable to distinguish between changes in income distributions for its property of transfer neutrality. So Pigou-Dalton transfer axiom may be a necessary but not sufficient condition to consider one measure as good to study economic inequality.

The numerical example, as cited above, may also be interpreted differently. It is a simple matter to verify that area covered by the income distributions q and r, and the egalitarian line is the same and consequently the Gini coefficient for these two different distributions also remains the same. It is an inherent property of this area-based measure and at times it may be misleading if an analyst makes a comment depending solely on this summary measure without supplementing her/his judgment by a visual examination of income distribution. The classical example that may be cited in support of the above-mentioned issue is that of 'Adanac'[2] as presented by Osberg (1981, p. 14; 2017) in connection with a study on economic inequality in Canada. It considers a simple two-class example in which the Gini coefficient is held constant, while the size of the rich and poor changes. It implies that in "Adanac', a series of different income distributions represent a constant Gini coefficient. According to the author (Osberg 2017), Canada offers as similar empirical example wherein between 2000 and 2011, Gini coefficient remains almost constant although income distributions (considering percentages of poor and rich in different strata) changed considerably. This constancy of the Gini coefficient has been very misleading to writers from popular press who made journalistic comments that since 2000 "inequality has not increased in Canada".[3] From this experience, the author feels that researchers should decide about which aspect of economic inequality matters the most – e.g., elite concentration or middle class inclusion or the share or income level of the disadvantaged – and supplement the use of any single summary measure of inequality (such as Gini coefficient) with direct examination of the relevant segment of the income distribution (Osberg 2017). In addition, there are references in literature to straightforward denial of the use of Gini coefficient as a summary measure (see Atkinson 1970, Piketty 2014, p. 266). According to the former, use of such summary statistics as the Gini coefficient "is misleading" for income distribution in developing countries as they are typically more equal at the bottom and less equal at the top than in the advanced countries. It implies that the use of Gini coefficient is inappropriate for left-leaning Lorenz curves[4,5]. According to the latter, such synthetic indices[6] "are sometimes useful, but they raise many problems. They claim to summarise in a single numerical index all that a distribution can tell us about inequality - …. This is very simple at a very first glance but inevitably somewhat misleading." Piketty (2014, p. 266) feels that one researcher may analyse inequalities in terms of distribution tables, indicating share of various groups in total income rather than using synthetic indices such as the Gini coefficient.

From the above, it is apparent that the use of Gini coefficient is inappropriate: (i) in cases of left-leaning Lorenz curves and (ii) when segment-wise analysis of inequality is a necessity. However, it appears that if the need for segment-wise analysis of inequality is ignored, the measure can be used robustly for Lorenz curves which are closer to the egalitarian line. The underlying weighting scheme of the Gini coefficient seems more appropriate for Lorenz curves which are closer to the egalitarian state. Any deviation from such a state due to transfers either at the lower or upper end may be considered to matter equally. For example, we consider a distribution s: (5.71, 6.67, 7.62, 8.57, 9.52, 10.48, 11.43, 12.38, 13.33, 14.29), and compute deviation of it from the egalitarian line as: (4.29, 3.33, 2.38, 1.43, 0.48, -0.48, -1.43, -2.38, -3.33, -4.29). We see that both the poorest and richest groups are equally dispersed from the egalitarian line (of course in different directions). One may check this criterion for other income groups too. In such a situation, one may treat any deviation in the condition of poor or rich equally. In reality, one may find nearly 30 of such income distributions in WIID 3.4 data (UNU-WIDER, 2017), which contains valid information of 4978 cases. However, if we envisage and progress towards a more equal society, Gini coefficient (or any equivalent one) will remain as the only appropriate apparatus for measuring economic inequality in days to come[7].

III Pro Transfer-sensitive Measures of Inequality

The Amato-Kakwani Inequality Index

As the measurement of inequality in today's highly unequal world remains somewhat unsatisfactory with the use of the Gini coefficient, there has been a recent revival of interest in a simple but attractive measure, which is nothing but the length of the Lorenz curve rescaled to range between 0 and 1. The higher the length of the Lorenz curve with respect to that of the egalitarian line, the more is the indication of the extent of deviation or inequality. It has been proposed first by Amato (1968, p. 261). Near about a decade later, Kakwani (1980, pp. 83-85) independently rediscovered the same and studied it in some detail. Arnold (2005, 2012), Majumder (2015), and Subramanian (2015) also discussed the measure with great interest. It may be called as Amato-Kakwani inequality index[8] and can be presented as follows:

$$AK=\frac{LC-\sqrt{2}}{2-\sqrt{2}} \qquad \ldots(1)$$

where the LC = length of the Lorenz curve that varies between √2 and 2.

Derivation of the working formula of it is also simple. When piece-wise linear (even when n is sufficiently large), Lorenz curve is the sum of all the hypotenuses of the right angled triangles beneath it. If we consider the share of income as x_i [i = 1, 2, …, n], arranged in non-decreasing order such that $\bar{x}$ or μ = $\sum x_i/n = 1/n$, as $\sum x_i = 1$, and proportion of population in each equally sized group as p = 1/n, an hypotenuse (say, h), will be:

$$h_i = \sqrt{x_i^2 + p^2}. \qquad \ldots(2)$$

As, $\bar{x}$ or μ = p = 1/n, after some manipulation of equation (2), we have:

$$h_i = \frac{1}{n}\sqrt{\left(\frac{x_i}{\bar{x}}\right)^2 + 1}. \qquad \ldots(3)$$

After taking summation in both sides of equation (3) for all i [i = 1, 2, …, n] and denoting the length of the Lorenz curve ($\sum h_i$) as LC, we get:

$$LC = \frac{1}{n}\sum_{i=1}^{n}\sqrt{\left(\frac{x_i}{\bar{x}}\right)^2 + 1}. \qquad \ldots(4)$$

Equation (4) may also be re-written as follows [denoting $\bar{x} = \mu$, as used in equation (2)]:

$$\mathrm{LC} = (1/n\mu)\sum_{i=1}^{n}\left(\mu^2 + x_i^2\right)^{\frac{1}{2}} \qquad \ldots(5)$$

Putting (5) in (1) we get the working formula of the said index as follows:

$$\mathrm{AK} = \frac{(1/n\mu)\sum_{i=1}^{n}(\mu^2+x_i^2)^{\frac{1}{2}}-\sqrt{2}}{2-\sqrt{2}} \qquad \ldots(6)$$

where, AK = Amato-Kakwani inequality index (or simply AK index).

We know from a lemma (Kakwani 1980, p. 67) that any inequality measure that is the arithmetic mean of a strictly convex function of income satisfies Pigou-Dalton transfer axiom. Equation (5) shows that L is the arithmetic mean of a strictly convex function of income which implies that the measure is sensitive to transfers at all levels of income. Equation (6) is equivalent to the one (numbered 5.35), as derived by Kakwani (1980, p. 84) for a continuous function:

$$\mathrm{L} = \frac{1}{(2-\sqrt{2})}\left[\frac{1}{\mu}\int_0^\infty \sqrt{\mu^2 + x^2}f(x)dx - \sqrt{2}\right] \qquad \ldots(7)$$

where L = 'A new inequality measure' based on the length of the Lorenz curve.

In regard to transfer-sensitivity property, Kakwani (1980, pp. 84-85) went a step further beyond Pigou-Dalton transfer axiom by investigating the sensitivity of the measure to transfers at different levels of income. He proved another lemma to show that the index (L) attaches higher weight to transfers at the lower end than at the middle and upper ends of a distribution. According to him, unlike the (area-based) Gini coefficient, this measure (based on the curve length) is more sensitive to transfers at the lower levels of income, making it particularly applicable to problems such as measuring the intensity of poverty.

Although AK index has attractive geometric interpretation (Arnold 2005) and desirable transfer-sensitivity properties (Kakwani 1980, pp. 84-85), its popularity up to now is very low. Authors like Arnold (2012) and Subramanian (2015), with an overt objective of making it popular, suggested easily comprehendible formulae of it. For example, according to Arnold (2012), undoubtedly (and partially) for the absence of a Lorenz curve free representation, the index is much less frequently employed. Consequently, in order to rectify 'this perceived fault', he suggested a simple representation of the index as an expectation of a particular convex function. Subramanian (2015) too proposes a working formula of the index which may be interpreted without drawing reference to the Lorenz curve. Equation (4) corresponds to the working formula of the index as proposed by Arnold (2012), the latter of which was derived for a discrete random variable (X) and which was of the form $\mathrm{A(X)} = \mathrm{E}(\sqrt{1 + (\mathrm{X/E(X)}^2}) = \mathrm{E}(\mathrm{g_A}(\mathrm{X/E(X)})$, where $g_A(X) = \sqrt{1 + x^2}$, a continuous convex function. Equation (6) corresponds to the equation (13) of

Subramanian (2015). The latter was derived using the same Lorenz-Gini apparatus[9].

Use of AK index with Numerical Example

AK index can be computed from micro data even when n is sufficiently large for all $x_i \geq 0$ [i = 1, 2, …, n][10]. However, as some authors suggested supplementing findings with visual examination of the income distributions, results are obtained from quintile as well as decile group-shares of income. We may consider the example cited in section II previously. The data and results are presented in Table 1 below.

An inequality measure (say, Z), which satisfies Pigou-Dalton transfer axiom will be transfer-neutral if Z(p) > Z(q) = Z(r); and Z will be pro transfer-sensitive if Z(p) > Z(r) > Z(q). In case of Gini coefficient, G(p) [= 0.330] > G(q) = G(r) [= 0.320]: the Gini coefficient is transfer-neutral. In case of AK index, AK(p) [= 0.101] > AK(r) [= 0.099] > AK(q) [= 0.094]: AK index is pro transfer-sensitive.

Table 1: Gini Coefficient and Amato-Kakwani Inequality Index[11]

Distribution	Q1	Q2	Q3	Q4	Q5	G	LC	AK
p	7	13	20	27	33	0.3300	1.4552	0.1009
q	9	11	20	27	33	0.3200	1.4523	0.0939
r	7	13	20	29	31	0.3200	1.4545	0.0993

Notes: Q: Quintile, G: Gini coefficient; LC: Length of the Lorenz curve; AK: Amato-Kakwani inequality index.
Source: Self-elaboration.

The above exercise shows that changes in different levels of an income distribution are captured well in the length of the Lorenz curve (LC) and consequently in the AK index. For a move from p to q, the value of AK drops from 0.1009 to 0.0939 (6.93 per cent decrease). For a similar move from p to r, the value of AK drops from 0.1009 to 0.0993 (1.98 per cent decrease). One may realise that drop in the said index is higher when comparatively poor people are benefitted. The results vividly support the spirit of Rawls' maximin rule where the claim of the worse-off of the two groups matters more. Results also support the claim of Kakwani (1980, p. 85) that unlike the Gini coefficient, the measure based on the curve length is more sensitive to transfers at the lower levels of income.

AK index has been considered as an alternative to Gini coefficient, as it is contented with all the essential properties of the latter with some more desirable properties. Its behaviour, in general (except sensitivity) and visual appeal in practice, does not differ much from those of the latter. User of Gini coefficient does not need a separate mind-setup to work with this. The scatterplot in figure 1 (in the appendix) shows how Gini coefficient and AK index go together following a particular pattern.

Subramanian's L index: The Left-wing Gini Coefficient

The second pro transfer-sensitive measure, as listed above, is Subramanian's L index. Conceptually, it works exactly in a similar fashion as Gini coefficient additionally with some more emphasis on transfers in the left-wing of the Lorenz curves. It has been derived by Subramanian (2015). He has shown that the transfer-neutral Gini coefficient can be presented as a linear (convex) combination of its two variants, the latter of which are anti transfer-sensitive and pro transfer-sensitive respectively. According to him, the latter, with pro transfer-sensitivity property, is reminiscent of a similarly 'left-wing' inequality measure, namely the 'Amato-Kakwani inequality index'. The existence of this measure is mentioned by Majumder (2015) and Osberg (2017), the latter of whom indicated a possible difficulty associated with the use of it for the absence of simple graphical representation as compared with that of the Gini coefficient. This 'left-wing' inequality measure is obtained in the same fashion of deriving the area-based Gini coefficient, and by definition it is a stand-alone measure favouring the left-leaning Lorenz curves. One does not require supplementing results of it by direct examination of the relevant segment of income distribution[12]. The index as derived by Subramanian (2015) is presented below[13]:

$$S_L = \left[\frac{2}{n(n-1)\mu}\right]\left[\sum_{i=1}^{n} i\{(\mu^2 + \mu_i^2)^{\frac{1}{2}} - \sqrt{2}\mu_i\}\right] \qquad \ldots(8)$$

where $\mu_i \equiv (1/i)\ \sum_{j=1}^{i} y_j$ and S_L = Subramanian's L index.

Use of the Subramanian's L index with Numerical Example

Using the same data, S_L is computed from formula (8) as presented in Table 2 below.

Table 2: Gini Coefficient and the Subramanian's L index

Distribution	Q1	Q2	Q3	Q4	Q5	G	S_L
p	7	13	20	27	33	0.3300	0.2643
q	9	11	20	27	33	0.3200	0.2539
r	7	13	20	29	31	0.3200	0.2567

Notes: Q: Quintile; G: Gini coefficient; S_L: Subramanian's L index.
Source: Self-elaboration.

We know that an inequality measure (say, Z), which satisfies Pigou-Dalton transfer axiom, will be pro transfer-sensitive if Z(p) > Z(r) > Z(q). In case of the Subramanian's L index, S_L(p) [= 0.2643] > S_L(r) [= 0.2567] > S_L(q) [= 0.2539]: the S_L is pro transfer-sensitive.

As S_L works in the same fashion of AK index, it supports the claim of Subramanian (2015) that the former is 'reminiscent' of the latter. However, they are not equivalent as the sensitivity of both differs. For example, for a move from p to q, the value of S_L drops from 0.2643 to 0.2539 (3.79 per cent decrease). For a similar move from p to r, the value of S_L drops from 0.2643 to 0.2567 (2.65 per cent decrease). When sensitivity of S_L is compared with the same of AK index (presented in the previous section), one may realise that (comparatively) the latter is more sensitive to the poor.

Conceptually, Subramanian's L index works exactly in a similar fashion as the Gini coefficient does, additionally with the property of pro transfer-sensitivity. Using the above-mentioned data-set, we have found that the index values of the two are perfectly correlated empirically as shown in figure 2 in the appendix.

Subramanian's L index is algebraically connected with the Gini coefficient (linearly) as follows (Subramanian 2015):

$$G = \left(\frac{1}{\sqrt{2}}\right) S_L + \left(1 - \frac{1}{\sqrt{2}}\right) S_R \qquad \ldots(9)$$

where S_R works as the Gini coefficient does, additionally with more emphasis on transfers in the right-wing of the income distributions. I would prefer it to be called as Subramanian's R index. Its formulation is shown below.

$$S_R = \left[\frac{2}{n(n-1)\mu}\right]\left[\frac{n(n+1)\mu}{\sqrt{2}(\sqrt{2}-1)} - \sum_{i=1}^{n} i\,\frac{(\mu^2+\mu_i^2)^{\frac{1}{2}}}{\sqrt{2}-1}\right] \qquad \ldots(10)$$

Studying S_R is beyond the objective of this paper. However, it may be mentioned that the measure is suitable in studies where the interest of the richer persons (at the upper end of income distribution) matters. In other words, it is anti transfer-sensitive. An inequality measure (say, Z), which satisfies Pigou-Dalton transfer axiom, will be anti transfer-sensitive if Z(p) > Z(q) > Z(r). In this case, it is a simple matter to verify that S_R(p) [= 0.4885] > S_R(q) [= 0.4796] > S_R(r) [= 0.4729]: the S_R is anti transfer-sensitive[14].

The Angle-based Pair of Inequality Measures

The angle-based inequality measure (a pair of measures actually) views the Lorenz curve or the egalitarian line as a ray of light. It envisages, with fantasy, that a society without economic inequality is nothing but a state or condition where light touches everybody without refraction (Majumder 2014). Literary, it considers the unit square as the society or economy and the egalitarian line as the passing of the ray of light without refraction. However, in reality we live in a stratified society with varying living conditions and the ray of light is perceived to refract every time it passes from one stratum to another forming the real-world Lorenz curve.

Such a concept is analogous to that of refraction of light of geometrical optics[15], where it measures bending of a ray of light passing from one homogeneous transparent medium to another. The extent of bending of a ray of light is measured by an index, namely the refractive index. We may also measure the extent of deviation of each segment of Lorenz curve following similar methodology, as demonstrated by Majumder (2015). Such an index may be called deviative index or index of deviation and I would prefer it to be called as Majumder's D index. When all such indices for different segments are added together and standardised, we have an overall measure for the whole income distribution. The latter (summary measure) is found to be exactly equivalent to the length-based AK index. As the perspective and methodology of deriving the overall measure under this approach differ from those of the AK index (in order to make a distinction between the two), we may call the said new summary measure as Majumder's AK index.

Refractive index measures the extent of bending of the ray of light, which is governed by Snell's law of the following form (Jenkins and White 1981):

$$m_a . \sin(\theta_a) = m_w . \sin(\theta_w) \qquad \ldots(11)$$

where, m_a is the refractive index of the medium a the light is leaving, θ_a is the angle of incidence, m_w is the refractive index of the medium w the light is entering, and θ_w is the angle of refraction. When we apply Snell's law in the Lorenz curve framework, each medium is considered as equivalent to one income group or stratum and the parameters in the left-hand side are assumed to be equivalent to those of the ideal condition, such as $m_a = 1.00$ and $\theta_a = 45^0$ (≡ the angle of incidence of the egalitarian line with respect to its vertical normal). Since we are interested in m_w for each income group or stratum, from equation (11) we derive:

$$m_w = \frac{\sin(45^0)}{\sin(\theta_w)} \qquad \ldots(12)$$

As $\sin(\theta_w) = p/\sqrt{x_i^2 + p^2}$ and $\sin(45^0) = 1/\sqrt{2}$, if we substitute these results in expression (12) and denote m_w as m (for the sake of simplicity), we have:

$$m_i = \frac{1}{\sqrt{2}} \frac{\sqrt{x_i^2 + p^2}}{p} \qquad \ldots(13)$$

where m_i is a measure of the extent of deviation of the i^{th} segment of the Lorenz curve with respect to the ideal condition. Conceptually, it is a measure of inequality for the particular segment of the Lorenz curve.

When $x_i = 0$ (share of income = 0), $m_{min} = 1/\sqrt{2} = 0.71$; when $x_i = p$ (share of income equals proportion of population), $m_{ideal} = (1/\sqrt{2})*\sqrt{2} = 1.00$; when $x_i = 1$ (one individual or group assumes all income), maximum value of m depends upon p. For example, when p = 0.2 (for n = 5) and $x_i = 1$, $m_{max} = \sqrt{(1.04)}/(0.2*\sqrt{2}) = 3.61$. In general, the maximum value of $m = (1/\sqrt{2})\sqrt{1+n^2}$.

As $\sqrt{x_i^2 + p^2} = h_i$ (where, h_i = segment of a Lorenz curve), equation (13) may also be written (symbolically) as follows:

$$m_i = n.\sin(45^0).h_i \qquad \ldots(14)$$

If we substitute the expression for h_i from equation (3) to equation (14), the working formula of this measure (deviation index) will be:

$$m_i = \frac{1}{\sqrt{2}}\sqrt{\left(\frac{x_i}{\bar{x}}\right)^2 + 1} \qquad \ldots(15)$$

If we take summation in both sides of equation (14) or (15) and denote $\sum m_i$ as M, we get:

$$M = k.LC \qquad \ldots(16)$$

where, $k = n.\sin(45^0)$ = constant, and by definition LC = length of the Lorenz curve as shown in equation (4) or (5). As M is nothing but the length of the Lorenz curve multiplied by a constant, when rescaled to range between 0 and 1, it becomes exactly equivalent to the AK index. It is needless to say that like LC in equation (5), M in equation (16) too, being the arithmetic mean of a strictly convex function of income, satisfies Pigou-Dalton transfer axiom. Now, we realise that M varies between $k\sqrt{2}$ ($\equiv$ n) and 2k ($\equiv n\sqrt{2}$). After rescaling, the overall index will take the following form (say, Majumder's AK index or AKM[16]):

$$AKM = \frac{R - k\sqrt{2}}{2k - k\sqrt{2}} = \frac{k.LC - k\sqrt{2}}{2k - k\sqrt{2}} = \frac{LC - \sqrt{2}}{2 - \sqrt{2}} \equiv AK \qquad \ldots(17)$$

The above equations (16 and 17) show the working formulae of the summary measure under the angle based approach.

Segment-wise Analysis of Inequality with Numerical Example

Although Arnold (2005, 2012) and Subramanian (2015) viewed AK index as a summary measure, AKM can be used to study inequality conditions separately for different segments of a distribution (as m_i). Using data from Table 1 the deviation index (m) and the overall measure (AKM) are computed and presented below.

Table 3: Deviation Index (m) and Majumder's AK (AKM): Hypothetical Examples

Distribution	Q1	Q2	Q3	Q4	Q5	G	G_L
p	7	13	20	27	33	0.3300	0.2643
q	9	11	20	27	33	0.3200	0.2539
r	7	13	20	29	31	0.3200	0.2567

Distribution	m_1	m_2	m_3	m_4	m_5	AKM
p	0.7492	0.8434	1.0000	1.1880	1.3643	0.1009
q	0.7754	0.8070	1.0000	1.1880	1.3643	0.0939
r	0.7492	0.8434	1.0000	1.2455	1.3043	0.0993

Notes: Q: Quintile group share; G: Gini coefficient, S_L: Subramanian's L index; m: Deviation index; AKM: The overall measure under the angle-based approach (Majumder's AK).

Source: Self-elaboration.

Table 4: Results of Pro Transfer-sensitive Measures of Inequality: One Real Example of Canada

	Country	Year	Source		D1	D2	D3	D4	D5
Data	Canada	2000	World Bank 2016		2.48	4.43	5.71	6.79	7.93
	Canada	2010	World Bank 2016		2.68	4.42	5.66	6.73	7.82
Result	Country	Year	AK	S_L	m_1	m_2	m_3	m_4	m_5
	Canada	2000	0.1164	0.2938	0.7285	0.7734	0.8143	0.8547	0.9025
	Canada	2010	0.1159	0.2935	0.7321	0.7731	0.8125	0.8523	0.8976

	Country	Year	Source		D6	D7	D8	D9	D10	DG
Data	Canada	2000	World Bank 2016		9.07	10.46	12.35	15.15	25.63	0.3653
	Canada	2010	World Bank 2016		8.97	10.44	12.31	15.23	25.74	0.3653
Result	Country	Year	AK	S_L	m_6	m_7	m_8	m_9	m_{10}	AKM
	Canada	2000	0.1164	0.2938	0.9546	1.0233	1.1237	1.2836	1.9454	0.1164
	Canada	2010	0.1159	0.2935	0.9499	1.0222	1.1215	1.2883	1.9526	0.1159

Notes: D: Decile group share; DG: Decile Gini coefficient (computed for n = 10); S_L: Subramanian's L index; AK: Amato-Kakwani index; m: Deviation index; AKM: The overall measure under the angle-based approach (Majumder's AK index).

Source: Self-elaboration of WIID3.4 (UNU-WIDER 2017) data.

Table 5: Inequality Indices according to Some Generalised Measures: Hypothetical Examples

Distribution	GE (-2)	GE (-1)	GE (0)	GE (1)	GE (2)	AT (0.5)	AT (1)
p	0.2482	0.1742	0.1359	0.1168	0.1090	0.0613	0.1271
q	0.1720	0.1387	0.1191	0.1087	0.1050	0.0557	0.1123
r	0.2474	0.1730	0.1342	0.1141	0.1050	0.0603	0.1255

Distribution	AT (2)	AT (4)	AT (10)	EG (2.5)	EG (3)	EG (4)	EG (10)
p	0.2584	0.4408	0.5816	0.3728	0.3960	0.4056	0.2042
q	0.2172	0.3548	0.4709	0.3571	0.3750	0.3760	0.1763
r	0.2571	0.4406	0.5816	0.3672	0.3930	0.4048	0.2042

Notes: GE: Generalised entropy measure, AT: Atkinson index; EG: Extended Gini coefficient; Number under parenthesis represents inequality aversion factor according to each formula.

Source: Self-elaboration.

As AK index and Majumder's AK index are equivalent, it is needless to say that the latter too is pro transfer-sensitive. Interpretation of m, which shows segment-wise inequality condition, is simple. An index value of 1.00 is desirable as it indicates an ideal condition represented by the egalitarian line. From the standpoint of income distribution, notionally an ideal condition is represented by the identity: $x_i = p$ (i.e., share of income equals proportion of population). Any index value of less than 1.00 is undesirable since it represents a condition where $x_i < p$ (i.e., share of income falls behind proportion of population). Interestingly, a refractive index value of less than 1.00 represents an 'anomalous' refraction in standard literature of optics (see Feynman 2011, p. 33-9) which makes ground to consider it as an 'anomalous' condition in economics too. An index value of more than 1.00 indicates a condition where: $x_i > p$ (share of income supersedes proportion of population). It indicates higher concentration of income or wealth which is also not desirable. The lowest value of m is 0.71. The highest values of m for n = 5 and n = 10 are 3.61 and 7.10 respectively[17].

Table 4 shows an example from Canada. The exercise is done in response to the questions raised by Osberg (2017) on journalistic comments on unchanged inequality there (discussed in section II) between 2000 and 2011[18]. From WIID3.4 (UNU-WIDER 2017) data we have captured two different distributions corresponding to years 2000 and 2010 respectively for which the Gini coefficient remains the same (0.3653 as computed from decile data). The upper portion of Table 4 shows data and its lower portion shows results. Although Gini coefficient remains the same, AK (or AKM) and S_L reveal differences between the distributions (AK decreased slightly from 0.1164 to 0.1159 while S_L decreased from 0.2938 to 0.2935). However, for segment-wise analysis, we need to rely on angle-based approach, i.e., on the deviation index (m). We may keep m_1 to m_6 in one group with m < 1.00 in the left, and m_7 to m_{10} to another group in the right with m > 1.00. We know that an increase in index value up to 1.00 in the left is desirable. We see that m_1 has increased over the years for good. At the same time, changes in m_2 to m_6 are undesirable, as values of the index decreased, which were already lying below 1.00. Similarly, we know that a decrease in index value to 1.00 in the right is desirable. We see that index values decreased for m_7 and m_8 for good. Concentration of income or wealth increased further in the two richest groups (as m_9 and m_{10} increased). However, based on the magnitude and importance (weight) of transfers in the left and right, the overall index (AK or AKM) shows a slight decline.

IV Generalised Inequality Measures

In order to address the issue of transfer neutrality of Gini coefficient, a number of generalised inequality measures have been proposed in the literature. While Gini coefficient has constant aversion to inequality, generalised inequality measures work with differing aversions to inequality. One researcher may set the value of the inequality aversion factor (as weight given either to the lower or upper end of an income distribution) as per the objective of the study (within the defined range of the respective measures). We have considered three generalised inequality indices for a comparison of their workability with the pro transfer-sensitive ones, such as: (i) Generalised entropy measure, (ii) Atkinson index of inequality, and (iii) Extended Gini coefficient. As the said indices are discussed widely in the literature, we are not going through the formulation details. Indices are computed using data presented in Table 1 and results are presented in Table 5. Results obtained from UNU-WIDER WIID 3.4 data are used in graphical analysis as shown in the appendix.

Generalised Entropy Measures

We begin with generalised entropy measure (GE). The inequality aversion factor of it (say α), varies from $-\infty$ to $+\infty$. When α is positive and large, GE is more sensitive to changes at the upper end of the income distribution. When α is negative, GE is more sensitive to changes at the lower end of the income distribution. The commonest values used for this inequality aversion factor are: -1, 0, 1 and 2 respectively. With $\alpha = 0$, the measure GE (0) is known as Theil's L (the mean log deviation measure); with $\alpha = 1$, the measure GE (1) is known as Theil's T (or Theil Index); and with $\alpha = 2$, the measure GE (2) becomes half the squared coefficient of variation. However, we have computed GE for α = -2, -1, 0, 1, and 2. Results using quintile data from Table 1 are presented in Table 5.

It is a simple matter to verify that GE (-2), GE (-1), GE (0) and GE (1) are pro transfer-sensitive; and GE (2) is transfer neutral. As the objective of the study is to consider pro transfer-sensitive measures only, we may concentrate on the first four variants of the GE measures only. Out of these four, the first three are not defined for the cases when income share for one (or more) in each is (are) equal to zero. The upper limit is also not defined for these three. The same for GE (1) is, however, known to be ln (n) [= 2.3026 for n = 10]. The actual minimum and maximum values of the said variants, as obtained from WIID3.4 (UNU-WIDER 2017) data, are presented below.

Table 6: Actual Minimum and Maximum Values of GE Measures as per UNU-WIDER Data

GE measures	Min. value	Max. value	n*
GE (-2)	0.0312	667.5797	4974
GE (-1)	0.0317	10.3742	4974
GE (0)	0.0330	1.5807	4974
GE (1)	0.0352	1.1688	4978

Notes: GE: Generalised Entropy measure, Figure in parenthesis represents inequality aversion factor according to each formula; and * Four cases became undefined for having zero share(s) of income under the first three measures.

Source: Self-elaboration of WIID3.4 (UNU-WIDER 2017) data.

One can guess from the above table that although comparison of inequality conditions is possible for more equal distributions, it is fairly impossible to do that when income distributions are highly unequal (as the upper limits sharply vary for most of them). So considering the practical concerns of exposition and the easiness of interpretation, one may remain with the GE (1), i.e., Theil's T as a pro transfer sensitive measure of inequality.

Four scatterplots are presented in the appendix to show how these generalised entropy measures go with the Gini coefficient. From figure 6 it is clear that one user of Gini coefficient does not need separate mind-setup in practice to go with Theil's T. However, the latter also does not qualify to be a good measure of economic inequality according to Piketty (2014, p. 266)[19].

Atkinson Index of Inequality

Atkinson index of inequality (AT index) is one of the most discussed measures in literature after the Gini coefficient. It ranges between 0 and 1. It works with an inequality aversion factor (say ε), where ε varies from 0 to ∞. One can choose appropriate value of ε to make it pro transfer-sensitive. According to the author of this measure, higher values of ε indicate more weight to transfers at the lower end of a distribution and (simultaneously) less weight to transfers at the top (Atkinson 1970). The recommended and the commonest used values of ε are: 0.5, 1, and 2 (Anand 1983, pp. 84-85). However, higher values of ε do not make the measure pro transfer-sensitive.

In Table 5, we have presented Atkinson index for five different values of ε: 0.5, 1, 2, 4, and 10. We understand that the first four variants of AT index are pro transfer-sensitive, as the condition AT(p) > AT(r) > AT(q) is fulfilled for each. One may notice that as the value of ε rises, AT index tends to ignore transfers at the upper end of income distributions (as the gap between the index values corresponding to distributions p and r decrease). AT (10) is the classical example of Rawlsian function $\min_i \{x_i\}$ as $\varepsilon \rightarrow \infty$, where interest of the poor only is considered ignoring completely the transfers among rich (Atkinson 1970; Anand 1983, p. 83). AT (10) for q is 0.4709 and the same of p and r is 0.5816. We know that p and r are two distinct distributions. So we cannot accept the same index value of inequality for these two[20]. This paper begins with the similar argument where the Gini coefficient has been the same (0.3200) for two distinct distributions such as q and r. We did not accept the result. Moreover, when $\varepsilon > 1$, AT index becomes undefined for zero share of income.

In figures 7 to 11, we have presented scatterplots Gini coefficient and different variants of AT index (to show how the relationship changes). Our objective is to choose one pro transfer-sensitive measure of economic inequality which works in a similar fashion as Gini coefficient does. Figure 7 shows that AT (0.5) and Gini coefficient closely go together. So one may choose AT (0.5) as a pro transfer-sensitive measure of economic inequality to work with[21].

Extended Gini Coefficient

Extended Gini coefficient (EG) works with an inequality aversion factor v which varies from 0 to ∞. When v = 2, EG becomes equivalent to the Gini coefficient. The weighting scheme is similar to that of AT index. Higher values of v attach more weight at the lower end and (simultaneously) less at the top. We have considered four values of v: 2.5, 3, 4, and 10. Results are displayed in Table 5 which also follow similar pattern as they do in the case of AT index. The first three are pro transfer-sensitive. GE (10) conveys the same as AT (10) does. However, if one looks at the figures 12 to 14, one may remain with the EG (2.5) as it and Gini coefficient closely go together[22].

Pro Transfer-sensitive Inequality Measures vs. Generalised Inequality Measures

From the above exercise, we have been able to pick up three generalised measures which can be employed as pro transfer-sensitive measures of inequality. They are: (i) Theil's T (i.e., generalised inequality measure with $\alpha = 1$), (ii) Atkinson inequality index with $\varepsilon = 0.5$, and (iii) Extended Gini coefficient with v = 2.5. However, selection of the value of the inequality aversion factor under each formula has been based on simple numerical exercises without much concentration on theoretical considerations. Also, those values of inequality aversion factor were selected for which we have found close association of each index with the Gini coefficient. They may vary over contexts and preferences of researchers. The pro transfer-sensitive measures presented in the previous section (such as, AK index or Majumder's AK index and Subramanian's L index) are seen more robust and free from biases of arbitrary selection of weights on transfers.

V A Comprehensive Summary of Characteristics of the Discussed Measures

Below is presented a comprehensive table which depicts characteristics of all the discussed measures of inequality. Although it is self-explanatory, the fourth column requires clarification. For example, in the case of Gini coefficient, as we have not come across any instance of segment-wise analysis of inequality so far, we have inserted the information 'not known'. As the Lorenz curve offers seemingly infinite number of possibilities to apply different tricks to derive different variants of Gini coefficient to address issues of our diverge interest (Subramanian 2015), the desirable property, which we are looking for may be recognised in the literature in due course. The same logic is applicable for many other measures. One should keep in mind that the concepts of 'decomposition' of an inequality measure and 'segment-wise analysis of inequality' are different. The former shows contribution of inequality 'within' and inequality 'between' sub-groups of the population to overall inequality. The latter shows (angle based) deviation of a particular segment of a Lorenz curve from the egalitarian line (measured by Majumder's D index). The final column of the Table has special significance as it shows our recommendation on the applicability of a particular measure (or else, as specified) for different types of Lorenz curve.

Table 7: Characteristics of All the Discussed Measures of Inequality

Measure	Transfer sensitive Property	Characteristics	Whether segment-wise analysis of inequality is possible	Remarks
Gini coefficient	Transfer-neutral	Equally sensitive to transfer between two poor individuals or groups and to transfer between two rich individuals or groups	Not known	Recommended for Lorenz curves, which are closer to the egalitarian line (interests of poor and rich matter equally)
The Amato-Kakwani inequality index (AK index)	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Yes, when derived as Majumder's AK index	Recommended for left-leaning Lorenz curves (when interest of the poor matters more)
Subramanian's L index: the left-wing Gini coefficient	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Recommended for left-leaning Lorenz curves (when interest of the poor matters more)
Subramanian's R index: the right-wing Gini coefficient	Anti transfer-sensitive	More sensitive to transfer between two rich individuals or groups than to transfer between two poor individuals or groups	Not known	Recommended for right-wing Lorenz curves (when interest of the rich matters more)
Majumder's AK index: the angle based summative measure	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Yes, as Majumder's D index (m)*: the angle based index of deviation of the each segment of the Lorenz curve	Recommended for left-leaning Lorenz curves (when interest of the poor matters more)

Contd…

Table 8: Characteristics of All the Discussed Measures of Inequality

Measure	Transfer sensitive property	Characteristics	Whether segment-wise analysis of inequality is possible	Remarks	
GE (α): Generalised entropy measure; α varies from $-\infty$ to $+\infty$	GE (-2)	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Undefined when one (or more) income share is zero
	GE (-1)	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Undefined when one (or more) income share is zero
	GE (0): Theil's L index	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Undefined when one (or more) income share is zero
	GE (1): Theil's T index	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Recommended for left-leaning Lorenz curves (when interest of the poor matters more)
	GE (2)	Transfer-neutral	Equally sensitive to transfer between two poor individuals or groups and to transfer between two rich individuals or groups	Not known	Recommended for Lorenz curves which are closer to the egalitarian line
AT (**ε**): Atkinson index; α varies from 0 to ∞	AT (0.5)	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Recommended for left-leaning Lorenz curves (when interest of the poor matters more)
	AT (1)	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Do not work in a similar fashion as Gini coefficient does
	AT (2)	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Do not work in a similar fashion as Gini coefficient does
	AT (4)	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Do not work in a similar fashion as Gini coefficient does
	AT (10)	Classical example of Rawlsian function $\min_i \{x_i\}$ as $\varepsilon \to \infty$	Interest of the poor only is considered	Not known	Hypothetical example
EG (v): Extended Gini coefficient; v varies from 0 to ∞	EG (2.5)	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Recommended for left-leaning Lorenz curves (when interest of the poor matters more)
	EG (3)	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Do not work in a similar fashion as Gini coefficient does
	EG (4)	Pro transfer-sensitive	More sensitive to transfer between two poor individuals or groups than to transfer between two rich individuals or groups	Not known	Do not work in a similar fashion as Gini coefficient does
	EG (10)	Classical example of Rawlsian function $\min_i \{x_i\}$ as $\varepsilon \to \infty$	Interest of the poor only is considered	Not known	Hypothetical example

Notes: *Analogous to the refractive index of geometrical optics; $m_{min} = 0.71$; $m < 1.00$ depicts anomalous condition; $m = 1.00$ depicts an ideal condition; $m > 1.00$ depicts higher concentration of wealth or income; m_{max} (for $n = 5$) = 3.61; m_{max} (for $n = 10$) = 7.11; in general, the maximum value of $m = (1/\sqrt{2})\sqrt{1+n^2}$.

VI Conclusion

Gini coefficient is transfer-neutral. It gives equal weight to transfers at both the lower and upper ends of an income distribution. Such a weighting scheme is more appropriate for studying Lorenz curves which are closer to the egalitarian state. Any deviation from such a state due to transfers either at the lower or upper end may be treated equally. Left-leaning Lorenz curves representing income distributions with higher concentration at the top (which is a reality for many in today's world), require a weighting scheme that gives more weight to transfers at the bottom. Such a weighting scheme conforms to the prime but latent objective of studying economic inequality too, which prioritises the claim of the worse-off of the two groups after any transfer. Left-leaning considerations in selection of appropriate inequality measure in today's world with high economic inequality are thus important. A study of the pro transfer-sensitive measures, in alternate to Gini coefficient (but within its legacy), is the resultant of such a synthesis. However, such measures are either much less frequently employed or less known. Some generalised measures are also considered, which can be tuned to the extent of pro transfer-sensitive ones, thanks to their differing aversions to inequality. Although these are widely discussed in the literature theoretically, their use is limited. In such a situation, this paper made an appraisal of the pro transfer-sensitive measures with the overt objective of making them popular. However, the emphasis was not on theoretical exaggeration, but on the practical concerns of exposition and on the advancement of easily comprehended and readily usable measures of inequality with desired properties.

Endnotes

1. When other things remain equal, its value declines with a progressive rank preserving transfer of income and vice versa.
2. If one reads 'Adanac' in reverse order, it will be spelled out as 'Canada'.
3. Similar statements appeared in British press too based on their experience as reported by Osberg (2017).
4. Which are skewed toward (1, 1) of the unit square so that they bulge at the bottom (see Subramanian 2015).
5. The measure he suggested (with two other generalised ones) is discussed in section 4.
6. Including Theil's entropy measure.
7. This is the reason for which we do not diverge much from that of the Gini coefficient and its legacy while exploring alternative measurement techniques.
8. Although in the recently developed literature, the measure is associated with the name of Amato and/or Kakwani, Lombardo (1969). Scala (1969) also discussed this measure as reported by Arnold (2005).
9. One may realise that the formula (13) of Subramanian (2015) has one typo. It needs correction such as replacement of the minus sign by a plus sign in the expression representing curve length as shown in the numerator of equation (6).
10. This is applicable for all the pro transfer-sensitive measures under discussion.

11. As n = 5, small-size correction is applied to both G and AK. For example, when n = 5, maximum curve length is 1.82, not 2.

12. Osberg (2017) made this general comment jointly for the angle based pair of inequality measures too as proposed by Majumder (2015). However, the comment is not appropriate as the said measures use the basic Lorenz curve framework for graphical representation and allow segment-wise analysis of inequality as demonstrated in sections 3.5 and 3.6.

13. Notations are usual, although Subramanian (2015) used absolute income levels instead of shares of income.

14. Values of G_R are not displayed in any table.

15. Or refraction of sound as studied in physical acoustics.

16. Although AK and AKM are equivalent, the latter readily allows segment-wise analysis of inequality.

17. For maximum value of m, please see discussion in connection with equation (13).

18. Osberg (2017) argued that "the income share of the top 1 per cent in Canada increased strongly" approximately from 2000 to 2008. However, as we are considering decile group shares of income, the present analysis will not answer exactly the question raised by Osberg (2017). It will rather show a procedure about how we can answer such questions using pro transfer-sensitive measures of inequality.

19. The criticism, however, gets void, as by definition it is a stand-alone measure favouring the left-leaning Lorenz curves. One does not require supplementing results of it by direct examination of the relevant segment of the income distribution.

20. Pigou-Dalton transfer axiom is also violated here.

21. Goodness of fit (with Gini coefficient) is even better in case of AT (0.25). However, for operational advantage use of AT (0.5) is advisable.

22. Goodness of fit (with Gini coefficient) is even better in case of EG (2.25). However, for operational advantage use of EG (2.5) is advisable.

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Annexure I

Figure 1: Scatterplot: Gini coefficient and AK index closely go together (n = 4978)

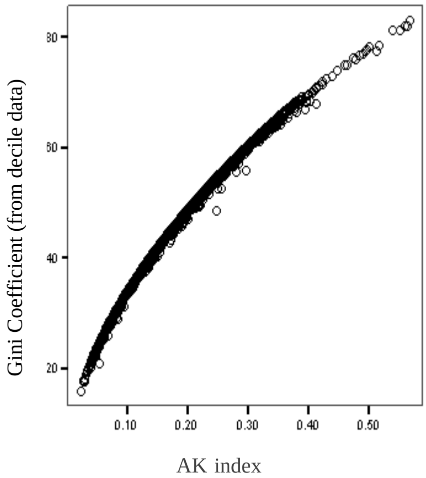

Figure 2: Scatterplot: Gini coefficient and Subramanian's L index go together (n = 4978)

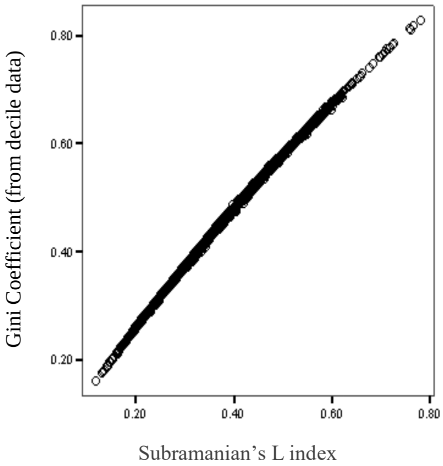

Figure 3: Scatterplot: Gini coefficient and GE (-2) do not go together (n = 4974)

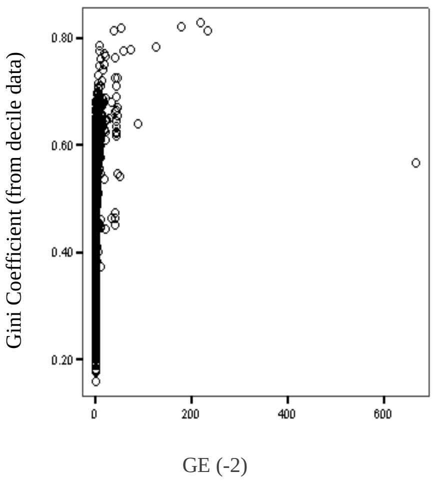

Figure 4: Scatterplot: Gini coefficient and GE (-1) do not go together (n = 4974)

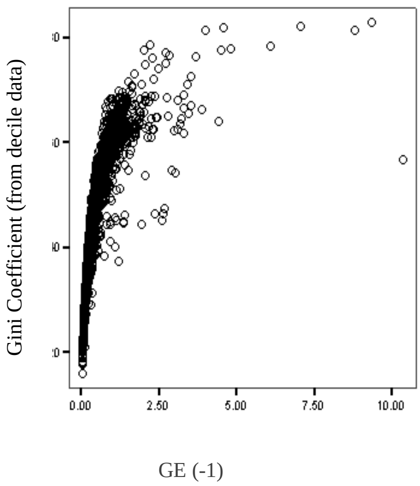

Figure 5: Scatterplot: Gini coefficient and GE (0) somewhat go together (n = 4974)

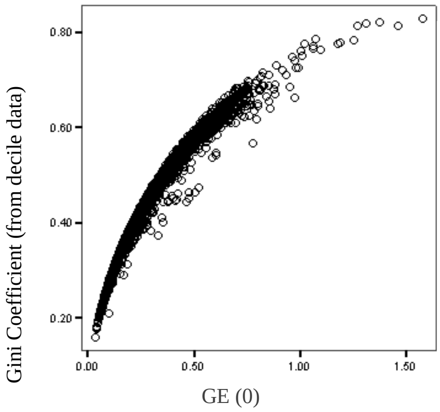

Figure 6: Scatterplot: Gini coefficient and GE (1) nearly go together (n = 4974)

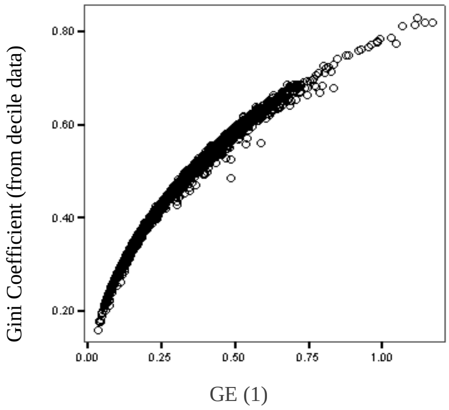

Figure 7: Scatterplot: Gini coefficient and AT (0.5) closely go together (n = 4978)

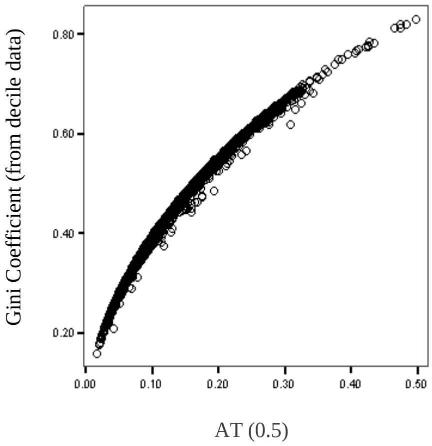

Figure 8: Scatterplot: Gini coefficient and AT (1) somewhat go together (n = 4978)

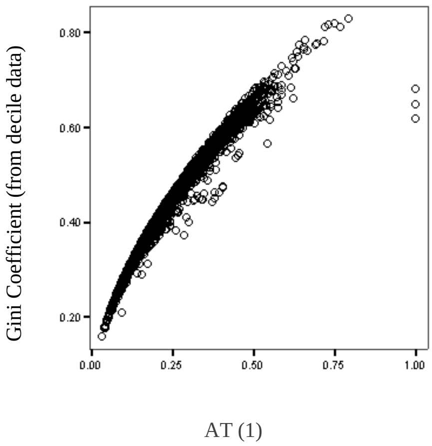

Figure 9: Scatterplot: Gini coefficient and AT (2) do not go together (n = 4974)

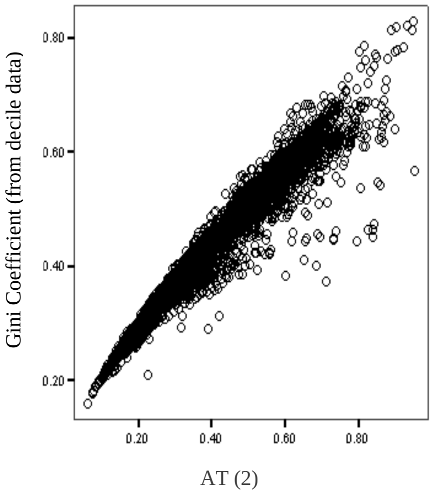

Figure 10: Scatterplot: Gini coefficient and AT (4) do not go together (n = 4974)

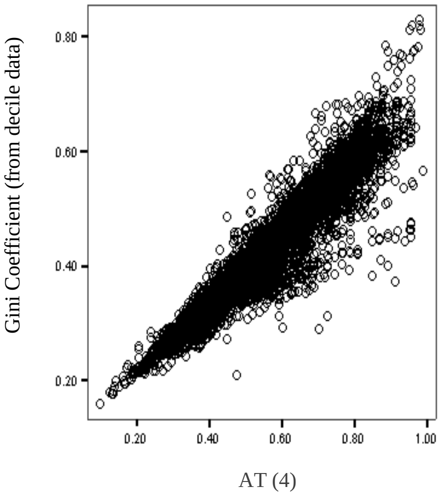

Figure 11: Scatterplot: Gini coefficient and AT (10) do not go together (n = 4974)

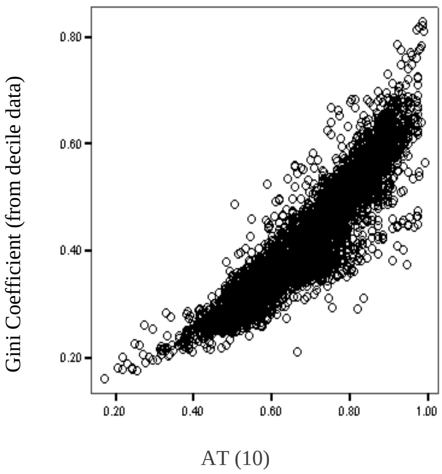

Figure 12: Scatterplot: Gini coefficient and EG (2.5) closely go together (n = 4978)

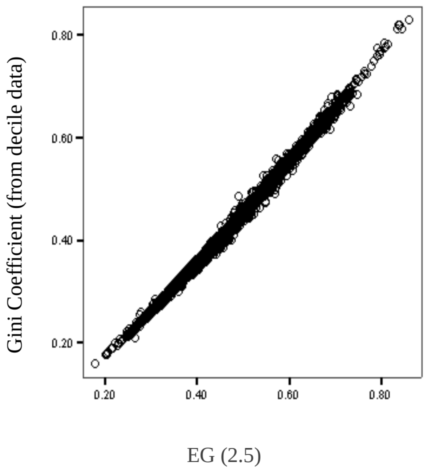

Figure 13: Scatterplot: Gini coefficient and EG (3) nearly go together (n = 4978)

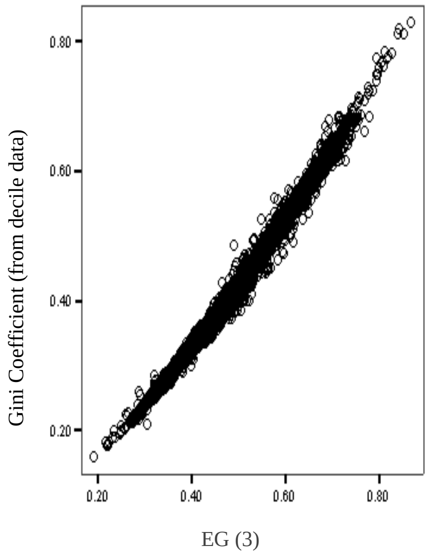

Figure 14: Scatterplot: Gini coefficient and EG (10) do not go together (n = 4978)