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ALTERNATIVE MEASURES OF ECONOMIC INEQUALITY[1]

AMLAN MAJUMDER[*]

1. Introduction

If we consider a particular distribution of income or consumption with n-number of groups / individuals, for same amount of transfer of resources between any two groups, Gini coefficient shows equal sensitivity provided transfer of income occurs between two successive groups / individuals. Such a judgment contradicts with the sensitivity level of our mind. We will not pay equal attention to a tiny transfer of resources between two persons in the worst off groups in a society and to a similar transfer between two persons in the best off groups in that society. Similarly, its sensitivity is constant for same amount of transfer from the worst off person to the best off person in the society and to the same in the reverse direction. Moreover, we can observe that Gini coefficient expresses more concern for countries, which are closer to the line of absolute equality. For example, according to the World Development Indicators 1999, Slovak Republic has more equal and Brazil has highly unequal distributions of income or consumption. According to that report (World Bank 1999), Gini coefficients for the two countries are 19.5 and 62.9 respectively[2]. However, it can be checked that for a tiny transfer of resources from one group to another, the changes in the Gini coefficient would be much higher in Slovak Republic than in Brazil. In order to address some of the above-mentioned issues, few other indices like variance, coefficient of variation and standard deviation of logarithms have also been brought into track, but those have been found incompetent either because of their total concentration on differences around mean or because of violating Pigou-Dalton condition. Pigou-Dalton condition implies that any transfer from a poorer person to a richer person, other things remaining the same, would always increase the inequality measure. In line with the same one may also think of decrease in inequality measure in response to transfers from the rich to the poor (Sen 1999). In such a situation it is necessary to develop some measures or modify the existing ones to address the above-mentioned issues. The present paper does similar exercise and develops some measures within the Lorenz curve framework or modifies the existing ones using available data set for 96 countries on distribution of income or consumption from World Development Indicators 1999.

[*] Dr. Amlan Majumder belongs to Economics Dept. Dinhata College, Cooch Behar.

2.1 The existing formulae

Though there are various ways to define the Gini coefficient, we will concentrate on the following two:

$$G_{KS} = (1/2n^2\mu)\sum_{i=1}^{n}\sum_{j=1}^{n}\left|y_i - y_j\right|, \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(i)}$$

$$G_S = 1 + (1/n) - (1/2n^2\mu)[y_1 + 2y_2 + \ldots + ny_n], \qquad \ldots \qquad \ldots \qquad \text{(ii)}$$

where, y_i is the income of person i, y_j is the income of person j, μ is the average level of income, i = 1, 2, 3, …, n, j = 1, 2, 3, …, n and $y_1 \geq y_2 \geq \ldots \geq y_n$. Equation (i) is due to Kendall and Stuart (1963), and equation (ii) is due to Sen (1973). In equation (i), Gini coefficient is one-half the average value of absolute differences between all pairs of incomes divided by the mean income. Equation (ii) shows income-waiting system in the welfare function behind the Gini coefficient, where the poorest person / group is weighted by n, the i^{th} person / group by (n + 1 - i), and the richest person / group by 1.

2.2. Modification of the formulae of Gini coefficient

Keeping the original spirit and essence unchanged, economists in field of measurement of inequality have always been in the quest of presenting simpler ways to calculate Gini coefficient. Among such efforts, the work of Pyatt, Chen, and Fei (1980), which has been used more recently by Lerman and Yitzhaki (1984), Yitzhaki (1994), and the works of Milanovic (1994, 1997) draw our attention. Among these, the first two have been operationalised mainly by taking into account covariance between income and ranks of all individuals. Milanovic (1994) has worked out a geometric formula with the intention of proposing an alternative and intuitively simpler derivation of the Gini coefficient. In his latter effort (see Milanovic 1997), he modifies the work of Pyatt, Chen, and Fei (1980) and claims that since all the components of the formula are easy to calculate, the Gini can be obtained using a simple hand calculator. Having similar objectives, in the present section we will try to modify the existing formulae of Gini coefficient as shown in equations (i) and (ii) in the previous section.

Anand (1997) has shown that the formulae given by Kendall and Stuart (1963) and Sen (1973) are same (i.e., $G_{KS} = G_S$ in section 2.1) for i = 1, 2, …, n; and j ≤ i. As both the measures are same, we will modify the G_{KS} (the formula given by Kendall and Stuart, 1963). The conditions of i = 1, 2, …, n; and j ≤ i make some operational advantage which restricts the study to all the elements of the lower triangular portion of the following symmetric matrix:

$$\begin{bmatrix} |y_1 - y_1| & |y_1 - y_2| & \cdots & |y_1 - y_n| \\ |y_2 - y_1| & |y_2 - y_2| & \cdots & |y_2 - y_n| \\ \cdots & \cdots & \cdots & \cdots \\ |y_n - y_1| & |y_n - y_2| & \cdots & |y_n - y_n| \end{bmatrix} \quad \ldots \quad \ldots \quad \ldots \quad \text{(iii)}$$

A light reasoning will reveal that such an operation includes $({}^nC_2 + n)$ numbers of elements, where C stands for 'combination'. It is clear that n numbers of diagonal terms of the above symmetric matrix $D = (d_{ij}),\ d_{ij} = |y_i - y_j|$ for all $i = j = 1, 2, \ldots, n$ are nothing but zeros, which do not reflect any inequality between two persons / groups. Presence of such elements in the numerator, and n^2 (n square) in the denominator of the Gini coefficient softens the results unnecessarily[3]. Those terms are, therefore, irrelevant for analysis and hence can be ignored. On the above background, one may confine the computation of Gini coefficient to ${}^nC_2$ numbers of elements, and make necessary adjustments in the existing formula.

In this paper as we are working with distribution of income or consumption, y_i or y_j is the proportion (share) of income or consumption of one particular group; and we assume that $y_1 \le y_2 \le \ldots \le y_n$. After modification, the equation (i) can be rewritten in general form as follows:

$$G = [1/(2\,{}^nC_2\mu)]\sum_{i=1}^{n}\sum_{j=1}^{n}|y_i - y_j|,$$

$$= [1/(n-1)]\sum_{i=1}^{n}\sum_{j=1}^{n}|y_i - y_j|, \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(iii)}$$

where C stands for 'combination' and the summations are over all combinations,

$$\mu = \frac{1}{n}\sum_{i-1}^{n} y_i = \frac{1}{n}, \text{ as } \sum_{i-1}^{n} y_i = 1, \text{ and } {}^nC_2 = \frac{n(n-1)}{2}.$$

As the data set consists of 5 different income groups (i = j = 5), after adjusting expression (iii) we get:

$$G = \frac{1}{4}\sum_{i=1}^{5}\sum_{j=1}^{5}|y_i - y_j| \quad \ldots \quad \ldots \quad \ldots \quad \text{(iv)}$$

It is to be noted that in order to normalise and standardise (to put in 0-1 scale as well) equations (i) and (ii), Kendall and Stuart (1963) and Sen (1973) have used the factor: 1/2μ. In the present exercise, as we have considered distribution of income or consumption, μ = 1/n = constant, for all distributions / countries. So, if we drop the factor 1/2μ from equation (iii), we have in general form:

$$G = [1/{}^{n}C_2)]\sum_{i=1}^{n}\sum_{j=1}^{n}\left|y_i - y_j\right|. \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(v)}$$

It is to be mentioned that equation (v) is the simplest expression (without any tedious adjustments) of Gini coefficient based on the straightforward logics within the Lorenz curve framework[4]. However, it will work well if we deal with distribution of income / consumption rather than considering absolute income / consumption levels. Also, being a measure of Gini coefficient, as it ranges between 0 and 0.4 (or 0 and 40 in 0-100 scale), it will be very difficult for us to comply with the results diverting from the mainstream spirit. So, we will confine our empirical exercise to equations (iii) and (iv) only.

Table 1 shows Gini coefficient (G) values for 96 countries, which have been computed using equation (iv). Though hypothetical minimum and maximum values are 0 and 1.00 respectively (without multiplying 100), actual minimum and maximum values are 0.227 (Slovak Republic) and 0.732 (Sieraleone) in table 1. Tables 2 to 6 show how sensitive the index is in response to 1 per cent transfer of resources from one group to another in upward and downward directions for few selected countries. Gini coefficient satisfies Pigou-Dalton condition. When reorganisation of income takes place from the worse off groups to the better off groups, value of G increases, and vice-versa. Within a country it is equally sensitive at all levels in both directions. For example, if we consider Australia we can see that for 1 per cent transfer of resources between two consecutive groups G changes by 1.267 per cent in both directions in tables 2 to 5. This is true for all other countries also. It means that sensitivity of Gini coefficient within a country or for a particular distribution is constant at all levels.

It directly follows from the property of the Gini index that equal (%) transfer of income between any two successive groups / individuals changes the Gini coefficient in the same manner in an economic system. But for different economic systems (countries), changes (%) may be different. In the present exercise, this is not due to differences in mean income, as we have considered distribution only (where μ = 1 / n for all the countries). However, among others, this may be due to joint impact of the inbuilt weighting system and share of income or consumption of different groups / individuals. After some straightforward simplification, the expression (iv) becomes:

$$(-1)y_1 + (-0.5)y_2 + (0)y_3 + (0.5)y_4 + (1)y_5 . \qquad \ldots \qquad \ldots \qquad \ldots \text{ (vi)}$$

It may be realised from the above that for a tiny transfer of resources between any two consecutive groups / individual (or between any two groups), changes may be higher in a country where shares at the lower ends are comparatively higher than in others. For example, when transfer of income (1 per cent) takes place between any two consecutive groups in Slovak Republic and Brazil, G changes by 2.203 and 0.739 per cent respectively in the two countries as shown in tables 2 to 5. In table 6, when reorganisation takes place between the best off (Q5) and worst off (Q1) groups, G changes by 8.811 and 2.954 per cent respectively in the two countries. It conveys that Gini coefficient has more concern for Slovak Republic than Brazil. If we compare income distribution of Australia and Belgium (from table 1), we see that the distribution is more equal in the latter than in the former. However, changes in Gini coefficient (in tables 2 to 5) are higher for Belgium than Australia. This can be checked for other countries also[5]. So, we can generalise the fact that Gini coefficient expresses more concern for countries, which are comparatively in better position or closer to the line of absolute equality. Though it pays equal attention to all sections of population within a country, when we see its performance across countries, it favours the well off ones.

2.3. Logarithmic transformation of Gini coefficient

Is it possible to make Gini coefficient more rational as we wish in terms of sensitivity by attaching more importance to transfer of resources at the lower ends? In order to check it, we have to take natural logarithm of income levels, and modify equations (iii) and (iv) as follows. In general form:

$$GL = \frac{1}{n-1}\sum_{i=1}^{5}\sum_{j=1}^{5} |\ln(y_i) - \ln(y_j)|$$

$$= \frac{1}{n-1}\sum_{i=1}^{5}\sum_{j=1}^{5} |\ln(y_i / y_j)| \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(vii)}$$

where ln = natural logarithm. For five different income groups / individuals:

$$G_L = \frac{1}{4}\sum_{i=1}^{n}\ \sum_{j=1}^{n} | \ln(y_i / y_j) | \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(viii)}$$

As G_L violets Pigou-Dalton condition (results are not displayed in tables), we should look for other suitable ways to fulfil our objective.

2.4. Geometric equivalent of Gini coefficient

We can develop a simple geometric formula to measure the area between the Lorenz curve and line of absolute equality. Such derivations are not new. Milanovic (1994) has derived one formula by looking at the vertical height between the 45 degrees line and the Lorenz curve. He has multiplied height of each strip by the corresponding population group to find the area of each strip, and taken summation over all population groups (n) to measure the whole area. In his formulation, the geometric Gini coefficient is equal to:

$$\sum_{j=1}^{n}\left(\sum_{i=1}^{j} p_i - \sum_{i=1}^{j} y_i\right) p_j, \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(ix)}$$

where p_i = proportion of recipients in the i^{th} group, y_i = proportion of total income received by the i^{th} group, and p_j = corresponding population group j. The above measure is a good example of alternative derivation of Gini-type functions for a continuous distribution in a discrete manner.

Use of various geometric applications for continuous and discrete frequency distributions can be found in the works of Anand (1997), who has reviewed several definitions and demonstrated their equivalence. Among these measures, the derivation using trapezium method is equivalent to one-half the relative mean deviation, a measure of inequality that has been discussed adequately by Sen (1973). However, the main problem with the relative mean deviation is that it does not always fulfil Pigou-Dalton condition. Other geometric measures as presented by Anand (1997) are based on simple geometric principles and are shown equivalent to the measure given by Kendall and Stuart (1963). In the quest of finding simpler and alternative derivations with robustness and accuracy, we will look forward to some other alternative geometric measures as appear below.

From figure 1 it is clear that the diagonal line has divided the rectangle into two equal triangles. For each triangle, base = height = 1.00 [as $\sum$ (proportions)=1.00], and the area is:

$$\frac{1}{2} \times base \times height = \frac{1}{2} \times 1 \times 1 = \frac{1}{2} \text{ unit.}$$

Area beyond the Lorenz curve is nothing but the sum area of *n* small triangles and (n-1) rectangles. In the first quintile, there is one triangle and in the rests, each group has one triangle and one rectangle. The area of each triangle is:

$$\frac{1}{2n} yi,$$

The sum area of *n* triangles will be:

$$\frac{1}{2n}, \text{ as } \sum_{i-1}^{n} y_i = 1.$$

Similarly, sum area of *(n-1)* rectangles is:

$$\frac{1}{n} y_1 + \frac{1}{n}(y_1 + y_2) + ... + \frac{1}{n}(y_1 + y_2 + ... + y_{n-1})$$

or, $$\frac{1}{n}[(n-1)y_1 + \cdots + \{n-(n-1)\}y_{n-1}]$$

or, $$\frac{1}{n}\sum_{j=1}^{n-1} y_j (n-j).$$

Total area beyond Lorenz curve is (sum area of triangles plus sum area of rectangles):

$$\frac{1}{2n} + \frac{1}{n}\sum_{j=1}^{n-1} y_j (n-j)$$

Area between the diagonal line and the Lorenz curve (≡ Gini coefficient) is:

$$\frac{1}{2} - \{\frac{1}{2n} + \frac{1}{n}\sum_{j=1}^{n-1} y_j (n-j)\}$$

or, $$\frac{(n-1) - 2\sum_{j=1}^{n-1} y_j (n-j)}{2n}$$

or, $$\frac{(n-1) - 2[(n-1)y_1 + \cdots + \{n-(n-1)\}y_{n-1}]}{2n}. \quad \dots \quad \dots \quad \dots \quad (x)$$

Expression (x) is one of the alternative geometric derivations of Gini coefficient. We may standardise the above expressions with the multiplication of 1/2μ or n/2 (as μ = 1/n):

$$G_G = \frac{(n-1)}{4} - \frac{1}{2}\sum_{j=1}^{n-1} y_j (n-j). \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(xi)}$$

For 5 individuals or groups the standardised expression will be:

$$G_G = \{1 - \frac{1}{2}(4y_1 + 3y_2 + 2y_3 + y_4)\}. \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(xii)}$$

It can be checked that Gini coefficient (G), and the geometric one (G_G) are identical with similar properties. The only difference is that: in case of G_G, minimum and maximum values range from -100 to +100 (after multiplying by 100). In case of G_G, if all resources are given to the poorest group / individual (Q1), G_G = - 100. In case of G, the maximum value is always 100. It can be understood that when all resources are given to Q1, the concept of Lorenz curve breaks. This fact is captured by G_G, but not by G (results are not displayed in tables). We will now check whether logarithmic transformation of it works well.

2.5. Logarithmic transformation of the Geometric coefficient

If we take natural logarithm of income levels in formula (xi) and (xii) then we have, in general form:

$$G_{GL} = \frac{(n-1)}{4} - \frac{1}{2}\sum_{j=1}^{n-1}(n-j)\ln(y_j), \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(xiii)}$$

and for 5 individuals / groups:

$$G_{GL} = [1 - \frac{1}{2}\{4\ln(y_1) + 3\ln(y_2) + 2\ln(y_3) + \ln(y_4)\}]. \qquad \ldots \qquad \text{(xiv)}$$

Hypothetical minimum and maximum values are 9.047 and ∝ (infinity). As we have adopted the mathematical operation of natural logarithm, if any group (at the extreme) has zero share to total income, the index value will tend to infinity. Conceptually, we have no problem with such results. Other conventional measures presuppose a maximum limit of inequality as 100 in 0-100 scale or 1 in 0-1 scale, where the worst condition is always comparable with other better conditions. However, as we are unable to compare a situation where people are dying without food with the one where people live like kings, an index value of ∝ (infinity) for such instance is not unjustifiable. Observed minimum and maximum values of G_{GL} are 10.449 (Slovak Republic) and 18.930 (Sieraleone) as shown in table 1. Logarithmic transformation of the geometric index served our purpose very well. It satisfies Pigou-Dalton condition. If we analyse its sensitivity, we can see that for 1 per cent transfer of income or consumption from Q1 to Q2, G_{GL} increases by

0.963 and 4.915 per cents in Slovak Republic and Brazil respectively. If share of income changes by 1 per cent from Q2 to Q3 in the two countries, the index values increase by 0.443 and 1.218 per cents respectively. For similar transfers between two consecutive groups in upward direction, the sensitivity of G_{GL} decreases gradually. It confirms that within a country the index is not equally sensitive at all levels. It always favours comparatively the worse off groups. If we compare the sensitivity levels of the G_{GL} in the two countries, we can see that those are much higher in Brazil than in Slovak Republic. It conveys that the index has more concern for Brazil or in general, for countries which are far from the line of absolute equality.

When transfer of income takes place in downward direction from Q5 to Q4, Q4 to Q3, and so on, sensitivity gradually increases (tables 4 and 5). However the extent of increase and decrease in both directions are not equal. In Brazil, for 1 per cent transfer of resources from Q1 to Q2 (in table 2), G_{GL} increases by 4.915 per cent. On the contrary, for 1 per cent transfer of resources from Q2 to Q1 (in table 5), G_{GL} decreases by 2.42 per cent. It tells that decrease in the level of welfare is more when the reorganisation of resources takes place in the upward direction and it is not completely recoverable by a similar transfer in the reverse direction.

2.6. The second area measure (equivalent to the geometric one)

We can go for another area measure based on cumulative proportion of income. If we assume that u_i's and v_i's are cumulative proportions of income on the line of absolute equality and on the Lorenz curve respectively then in general (and standardised) form:

$$G_A = \frac{1}{2n\mu}\sum_{i=1}^{n}(u_i - v_i)$$

$$= \frac{1}{2}\sum_{i=1}^{n}(u_i - v_i)\,. \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(xiv)}$$

For 5 individuals / groups:

$$G_A = \frac{1}{2}\sum_{i=1}^{5}(u_i - v_i)\,. \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(xv)}$$

It can be checked that G_A and G_G are identical with similar properties. So, it is needless to study its properties separately. However, we will check whether logarithmic transformation of it could be a good substitute of G_{GL}.

2.7. Logarithmic transformation of the second area measure

We can take natural logarithm of u_i's and v_i's and modify formula (xiv) and (xv) as follows:

$$G_{AL} = \frac{1}{2}\sum_{i=1}^{n} \ln(u_i / v_i), \qquad \dots \qquad \dots \qquad \dots \qquad \text{(xvi)}$$

for n-number of groups / individuals; and

$$G_{AL} = \frac{1}{2}\sum_{i=1}^{5} \ln(u_i / v_i), \qquad \dots \qquad \dots \qquad \dots \qquad \text{(xvii)}$$

for 5 different groups / individuals.

Hypothetical minimum and maximum values are 0 and ∝ (infinity). However, if all resources are given to Q1 the index may take a value of – 1.630. Observed minimum and maximum values are 0.646 (Slovak Republic) and 3.888 (Sieraleone) in table 1. If we compare sensitivity of G_{AL} and G_{GL} (in tables 2-5) we can see that the former is more sensitive at all levels than the latter. Another added advantage of using G_{AL} could be found from table 6. For 1 per cent transfer from Q1 to Q5 in Slovak Republic, G_{GL} increases by 1.68 per cent, and G_{AL} increases by 12.449 per cent. The figures for Brazil are 6.444 and 12.812 respectively. G_{GL} makes judgement by looking at the condition of the better off groups. As income has diminishing marginal utility (and hence we have taken natural logarithm of it), a tiny transfer from the poorest to the richest group will hardly increases the level of welfare of the latter; and consequently the level of inequality in the society. On the contrary, G_{AL} makes judgement by looking at the condition of the worse off groups. Income may have diminishing marginal utility but by losing resources poor people suffer more and it increases the overall inequality in the society.

2.8. Trigonometric measures

So far, we have come across arithmetic and geometric derivations of Gini coefficient, which directly or indirectly focus on the area between the line of absolute equality and the Lorenz curve. In this section, we will look at the different complementary angles of the right-angled triangles formed by (and below) the Lorenz curve. This shift of focus from area to functions based on the relationships between sides and angles of triangles is very simple, but epistemologically equally important to similar endeavours made by our ancestors long past in different branches of mathematics to explain complex phenomena. So, in the quest of alternative and simpler derivations we are adopting trigonometric applications in measures of economic inequality as appear below.

We know that there are n-numbers of right-angled triangles below the Lorenz curve corresponding to n-numbers of individuals / groups. For each triangle, we can compute cosecant or cotangent and add them to get a measure of inequality. By looking at the left-hand side complementary angle of each right-angled triangle, we may measure cotangent of it, which is nothing but the base of the triangle divided by perpendicular of it. The trigonometric measures based on cotangent (of left-hand side complementary angle) of a triangle are as follows (in general form):

$$G_{CT} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i}.$$

We will further modify and standardise it by subtracting n from it as well as multiplying it by n/2 as follows:

$$G_{CT} = \frac{1}{2}\left(\sum_{i=1}^{n}\frac{1}{y_i} - n^2\right). \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(xviii)}$$

For 5 individuals / groups:

$$G_{CT} = \frac{1}{2}\left(\sum_{i=1}^{n}\frac{1}{y_i} - 25\right). \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(xix)}$$

Cosecant of a triangle (of left-hand side complementary angle) is the hypotenuse divided by perpendicular of it. The measure based on it is as follows (in general form):

$$G_{Co\sec} = \sum_{i=1}^{n}\frac{x_i}{y_i},$$

where $x_i = \sqrt[2]{(1/n)^2 + y_i^2}$ = hypotenuse of each triangle.

After some manipulation and standardisation (deducting $n\sqrt{2}$ and multiplying by n/2) we have:

$$G_{Co\sec} = \frac{n}{2}\left(\sum_{i=1}^{n}\frac{x_i}{y_i} - n\sqrt{2}\right). \qquad \ldots \qquad \ldots \qquad \ldots \qquad \text{(xx)}$$

The rationale behind computing G_{CT} and G_{Cosec} is quite interesting. In G_{CT} cotangent of left-hand side complementary angle of each triangle is:

$$\frac{1}{ny_i},$$

where 1/n is the base (= constant), and $1/y_i$ is the perpendicular of the triangle. The hypotenuse is nothing but the portion of the Lorenz curve. We can realise that when the distribution is highly unequal (meaning flatter Lorenz curve or portions of it or hypotenuses of the triangles, and smaller complementary angles in the left-hand side), at the initial stages y_i's will be smaller leading to larger cotangent values. As y_i's tend to increase, cotangent values tend to decrease. G_{Cosec} also works in a similar fashion, where for a highly unequal distribution cosecant values tend to be larger initially. As y_i's, and hence x_i's tend to increase, cosecant values tend to decrease[6]. So, even if we do not measure area between the Lorenz curve and the line of absolute equality, we see that the relationships between sides and angles of the triangles below the Lorenz curve brilliantly reflect the inequalities embedded in the distributions.

It can be checked that G_{Cosec} and G_{CT} work almost similarly with equal sensitivity at different levels. As computation of G_{CT} is comparatively easier, we will study properties of G_{CT} only. Hypothetical minimum and maximum values of G_{CT} are 0 and $\propto$ (infinity). However, like the logarithmic measures, if share of any group becomes 0, value of G_{CT} will tend to infinity. Observed minimum and maximum values are 1.370 (Slovak Republic) and 65.955 (Sieraleone) in table 1. If we look at sensitivity, we can see that it is more sensitive than any other index as presented above. For example, for 1 per cent transfer from Q1 to Q2, the index increases by 14.383 per cent in Slovak Republic, 48.239 per cent in Brazil, and 676.541 per cent in Sieraleone as shown in table 2 (Sieraleone has the worst distribution of income or consumption as shown in table 1). Almost similar (slightly higher) changes can be observed when transfer of income takes place from Q1 to Q5. The percentage figures are 24.542 for Slovak Republic, 53.444 for Brazil, and 689.157 for Sieraleone (table 6).

It is to be noted that, in the case of Sieraleone in table 5, after reorganisation of resources from Q2 to Q1, G_{CT} and G increase by 5.087 and 0.068 per cent respectively. In this case, after 1 per cent transfer of resources in downward direction, the condition: $y_1 \leq y_2 \leq \ldots \leq y_n$ is violated. In fact, after reorganisation, Q1 benefited; but the condition of Q2 deteriorated and its level reached below that of Q1, and hence inequality increased in one sense. If we transfer, say, 3 or 4 per cent resources from Q2 to Q1 in the countries, we will have many such instances. It is to be kept in mind that Pigou-Dalton condition requires that transfer must be rank-preserving. G_{GL} and G_{AL} also satisfy the condition, but somewhat partially or weakly ignoring the rank-order condition: $y_1 \leq y_2 \leq \ldots \leq y_n$. However, Gini coefficient (G) and the trigonometric one (G_{CT}) satisfy the same condition very strictly. The other two of the three basic properties that one would like an inequality index to satisfy are: mean or scale independence, and population-size independence (see Anand 1997). These two conditions require that if everyone's income is changed in the same proportion, and similarly if number of people at each income level is changed by the same proportion, the index remains invariant. We may realise that all the measures as discussed and / or derived above satisfy these two conditions, as instead of considering absolute income levels or size of population we have considered distributions. So, all the alternative derivations except the logarithmic ones strictly satisfy all the three properties of an ideal inequality measure.

3. Conclusion

One appeal of the Gini coefficient, as claimed by Sen (1999), is that it takes note of differences between every pair of incomes. In formulae (i) and (ii) number of pairs of difference is n^2 (n square). However, we have adjusted the formulae (considering all possible combinations) by reducing the number of pairs to ${}^nC_2$. After doing this adjustment too, we find that Gini coefficient is identical with other geometric or area measures. Even if we do not count differences between all possible combinations (or pairs), results are unaffected, as we have seen that G_A, G_G, and G are identical. Income-waiting system in the welfare function behind the formula (ii) given by Sen (1973) and the derived formula (x) following simple geometrical procedure is similar. However, the same of formula (iv) as shown in expression (vi) is quite typical. Logarithmic transformation of Gini coefficient fails. The same of other similar or identical measures worked well ignoring the rank-order condition. If we compare the performance of G_{GL} and G_{AL}, we can realise that G_{AL} is better. Trigonometric measures are not supposed to initiate any debate as they are not very direct and also the quandary of logarithmic transformation of income levels in those is absent. Also, G_{CT} worked very well strictly following all the three properties of an ideal inequality measure. If we take into account all these things and look at the degree of sensitivity, G_{CT} is the best and simplest measure among all within the periphery of Lorenz curve framework.

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[1] I gratefully acknowledge comments by the anonymous referee.

[2] In the present exercise, the computed figures (using modified formula) are 22.7 and 67.9 for Slovak Republic and Brazil respectively as shown in table 1.

[3] For lenient reasoning one may recall the results of Slovak Republic and Brazil as presented in section 1, and compare those with the same in the endnote 2.

[4] Derivation of this formula is important, as it is identical with other alternative geometric measures (without standardisation) in the subsequent sections.

[5] Though in order to save space we have not displayed results of all other countries.

[6] We must keep in mind that for highly unequal distributions, y_i's and x_i's change very slowly at the lower ends, and jump suddenly at the upper ends. The extent of change is higher in the former than in the latter.

Table 1. Distribution of income or consumption and different measures of Inequality

Country	Distribution of Income or Consumption*					Measures of Inequality			
	Q1	Q2	Q3	Q4	Q5	G	G_{GL}	G_{AL}	G_{CT}
Algeria	0.070	0.116	0.161	0.227	0.426	0.412	12.118	1.348	5.435
Australia	0.070	0.122	0.166	0.233	0.409	0.395	11.998	1.301	5.122
Austria	0.104	0.148	0.185	0.229	0.333	0.270	10.817	0.809	2.074
Bangladesh	0.094	0.135	0.172	0.220	0.379	0.328	11.250	0.985	3.022
Belarus	0.085	0.135	0.177	0.231	0.372	0.335	11.398	1.054	3.420
Belgium	0.095	0.146	0.184	0.230	0.345	0.292	11.022	0.898	2.528
Bolivia	0.056	0.097	0.145	0.220	0.482	0.488	12.952	1.684	8.342
Brazil	0.025	0.057	0.099	0.177	0.642	0.677	15.853	2.833	24.926
Bulgaria	0.083	0.130	0.170	0.223	0.393	0.357	11.560	1.119	3.826
Burkina Faso	0.055	0.087	0.120	0.187	0.550	0.545	13.422	1.867	10.088
Canada	0.075	0.129	0.172	0.230	0.393	0.369	11.748	1.200	4.396
Chile	0.035	0.066	0.109	0.181	0.610	0.633	14.853	2.442	17.531
China	0.055	0.098	0.149	0.223	0.475	0.483	12.939	1.680	8.343
Colombia	0.031	0.068	0.109	0.176	0.615	0.638	15.065	2.528	19.223
Costa Rica	0.040	0.088	0.137	0.217	0.518	0.543	13.835	2.036	12.601
Cote d Ivory	0.068	0.112	0.158	0.222	0.441	0.428	12.258	1.403	5.868
Czech R	0.105	0.139	0.169	0.213	0.374	0.306	11.019	0.879	2.502
Denmark	0.096	0.149	0.183	0.227	0.345	0.288	10.982	0.881	2.448
Dominican R	0.042	0.079	0.125	0.197	0.557	0.574	14.039	2.119	13.170
Ecuador	0.054	0.089	0.132	0.199	0.526	0.527	13.298	1.821	9.628
Egypt	0.087	0.125	0.163	0.214	0.411	0.369	11.588	1.122	3.868
El Salvador	0.037	0.083	0.131	0.205	0.544	0.568	14.152	2.162	14.213
Estonia	0.062	0.120	0.170	0.231	0.418	0.412	12.246	1.404	6.033
Ethiopia	0.071	0.109	0.145	0.198	0.477	0.451	12.356	1.436	6.151

Finland	0.100	0.142	0.176	0.223	0.358	0.299	11.021	0.890	2.501
France	0.072	0.127	0.171	0.228	0.401	0.380	11.863	1.248	4.745
Gambia The	0.044	0.090	0.135	0.204	0.528	0.541	13.656	1.967	11.521
Germany	0.090	0.135	0.175	0.229	0.371	0.328	11.300	1.010	3.148
Ghana	0.084	0.122	0.158	0.219	0.417	0.382	11.714	1.174	4.197
Guatemala	0.021	0.058	0.105	0.186	0.63	0.673	16.092	2.914	28.174
Guinea	0.064	0.104	0.148	0.212	0.472	0.462	12.579	1.532	6.916
Guinea-Bissau	0.021	0.065	0.120	0.206	0.589	0.639	15.737	2.761	26.445
Guyana	0.063	0.107	0.150	0.212	0.469	0.459	12.554	1.523	6.867
Honduras	0.034	0.071	0.117	0.197	0.580	0.609	14.688	2.376	16.922
Hungary	0.097	0.139	0.169	0.214	0.381	0.322	11.175	0.950	2.859
India	0.092	0.130	0.168	0.217	0.393	0.345	11.380	1.036	3.334
Indonesia	0.080	0.113	0.151	0.208	0.449	0.417	11.998	1.286	5.003
Ireland	0.067	0.116	0.164	0.224	0.429	0.416	12.193	1.380	5.719
Israel	0.069	0.114	0.163	0.229	0.425	0.414	12.156	1.363	5.560
Italy	0.076	0.129	0.173	0.232	0.389	0.365	11.711	1.185	4.286
Jamaica	0.058	0.102	0.149	0.216	0.475	0.474	12.789	1.619	7.746
Jordan	0.059	0.098	0.139	0.203	0.501	0.495	12.915	1.667	8.135
Kazakhstan	0.075	0.123	0.169	0.229	0.404	0.382	11.839	1.235	4.611
Kenya	0.050	0.097	0.142	0.209	0.502	0.508	13.226	1.796	9.564
Kyrgyz R	0.067	0.115	0.164	0.231	0.423	0.414	12.191	1.379	5.706
Lao PDR	0.096	0.129	0.163	0.210	0.402	0.347	11.353	1.018	3.277
Latvia	0.083	0.138	0.180	0.229	0.370	0.333	11.400	1.057	3.460
Lesotho	0.028	0.065	0.112	0.194	0.601	0.638	15.260	2.597	20.923
Lithuania	0.081	0.123	0.162	0.213	0.421	0.385	11.763	1.197	4.359
Luxembourg	0.095	0.136	0.177	0.224	0.367	0.316	11.180	0.958	2.859
Madagascar	0.051	0.094	0.133	0.201	0.521	0.524	13.318	1.832	9.830
Malaysia	0.046	0.083	0.130	0.204	0.537	0.552	13.727	1.994	11.622
Mali	0.046	0.080	0.119	0.193	0.562	0.573	13.898	2.061	12.302
Mauritania	0.062	0.108	0.154	0.220	0.456	0.450	12.528	1.514	6.810
Mexico	0.036	0.072	0.118	0.192	0.582	0.606	14.557	2.325	16.034
Moldova	0.069	0.119	0.167	0.231	0.415	0.402	12.063	1.327	5.311
Mongolia	0.073	0.122	0.166	0.230	0.409	0.390	11.921	1.269	4.856
Morocco	0.066	0.105	0.150	0.217	0.463	0.453	12.478	1.490	6.555
Nepal	0.076	0.115	0.151	0.210	0.448	0.420	12.069	1.320	5.235
Netherlands	0.080	0.130	0.167	0.225	0.399	0.367	11.647	1.154	4.066
Nicaragua	0.042	0.080	0.126	0.200	0.552	0.570	14.005	2.106	13.029
Niger	0.026	0.071	0.139	0.231	0.533	0.587	14.973	2.464	20.473
Nigeria	0.040	0.089	0.144	0.234	0.494	0.527	13.731	1.992	12.239
Norway	0.100	0.143	0.179	0.224	0.353	0.294	10.991	0.879	2.438
Pakistan	0.094	0.130	0.160	0.203	0.412	0.355	11.419	1.046	3.467
Panama	0.023	0.062	0.113	0.198	0.604	0.649	15.706	2.762	25.081
Papua NG	0.045	0.079	0.119	0.192	0.565	0.577	13.963	2.088	12.631
Paraguay	0.023	0.059	0.107	0.187	0.624	0.665	15.863	2.829	25.862
Peru	0.044	0.091	0.141	0.213	0.512	0.529	13.575	1.934	11.228
Philippines	0.059	0.096	0.139	0.211	0.496	0.495	12.927	1.671	8.158
Poland	0.093	0.138	0.177	0.226	0.366	0.317	11.196	0.966	2.903
Romania	0.089	0.136	0.176	0.226	0.373	0.329	11.312	1.016	3.188
Russian F	0.042	0.088	0.136	0.207	0.528	0.546	13.768	2.011	12.125
Rwanda	0.097	0.132	0.165	0.216	0.391	0.336	11.272	0.986	3.066
Senegal	0.031	0.074	0.122	0.195	0.579	0.609	14.774	2.406	17.912
Sieraleone	0.011	0.020	0.098	0.237	0.634	0.732	18.930	3.888	65.955
Slovak R	0.119	0.158	0.188	0.222	0.314	0.227	10.449	0.646	1.370
Slovenia	0.093	0.133	0.169	0.219	0.386	0.336	11.314	1.010	3.173
South Africa	0.029	0.055	0.092	0.177	0.648	0.680	15.683	2.768	22.864
Spain	0.075	0.126	0.170	0.226	0.403	0.378	11.803	1.221	4.529
Sri Lanka	0.089	0.131	0.169	0.217	0.393	0.347	11.429	1.060	3.470

Sweden	0.096	0.145	0.182	0.232	0.345	0.293	11.018	0.895	2.508
Switzerland	0.074	0.116	0.156	0.219	0.435	0.413	12.056	1.318	5.205
Tanzania	0.068	0.110	0.152	0.216	0.455	0.440	12.338	1.434	6.102
Thailand	0.056	0.087	0.130	0.200	0.527	0.528	13.273	1.807	9.471
Tunisia	0.059	0.104	0.153	0.221	0.463	0.463	12.688	1.579	7.393
Turkmenistan	0.067	0.114	0.163	0.228	0.428	0.418	12.217	1.389	5.777
Uganda	0.066	0.109	0.152	0.213	0.461	0.447	12.418	1.467	6.384
Ukraine	0.043	0.090	0.138	0.208	0.522	0.538	13.671	1.972	11.668
UK	0.071	0.128	0.172	0.231	0.398	0.379	11.867	1.249	4.776
USA	0.048	0.105	0.160	0.235	0.452	0.469	13.010	1.709	9.037
Venezuela	0.043	0.088	0.138	0.213	0.518	0.538	13.692	1.981	11.746
Vietnam	0.078	0.114	0.154	0.214	0.440	0.412	12.001	1.291	5.016
Yemen R	0.061	0.109	0.153	0.216	0.461	0.454	12.562	1.529	6.951
Zambia	0.042	0.082	0.128	0.201	0.548	0.566	13.950	2.084	12.809
Zimbabwe	0.040	0.063	0.100	0.174	0.623	0.639	14.762	2.401	16.613

* Source: World Development Indicators 1999

Q: Quintile, G: Gini coefficient, G_{GL}: Logarithmic transformation of the geometric index, G_{Al}: Logarithmic transformation of the second area measure, G_{CT}: The trigonometric measure based on cotangent of left-hand side complementary angle of all triangles below the Lorenz curve.

Table2. Sensitivity of different measures of inequality*

Country	1 % transfer from Q1 to Q2				1 % transfer from Q2 to Q3			
	G	G_{GL}	G_{AL}	G_{CT}	G	G_{GL}	G_{AL}	G_{CT}
Australia	1.267	1.585	5.922	17.182	1.267	0.582	2.055	3.803
Belgium	1.712	1.117	6.193	15.807	1.712	0.485	2.360	4.419
Brazil	0.739	4.915	9.015	48.239	0.739	1.218	2.295	5.629
India	1.451	1.045	5.553	11.641	1.451	0.547	2.224	4.599
Sieraleone	0.684	22.121	30.833	676.541	0.752♣	4.979	5.008	37.188
Slovak Republic	2.203	0.799	6.790	14.383	2.203	0.443	2.844	5.801
USA	1.066	2.542	6.836	25.750	1.066	0.688	1.978	3.512

* Different column shows how the values of the indices change in response to 1 per cent transfer of resources between two consecutive groups.

♣ Exceptional case where % change ≠ constant (as after transfer the condition: $y_1 \leq y_2 \leq \ldots \leq y_n$ is violated and it can be checked from the distribution in table 1). Q: Quintile, G: Gini coefficient, G_{GL}: Logarithmic transformation of the geometric index, G_{Al}: Logarithmic transformation of the second area measure, G_{CT}: The trigonometric measure based on cotangent of left-hand side complementary angle of all triangles below the Lorenz curve.

Table3. Sensitivity of different measures of inequality*

Country	1 % transfer from Q3 to Q4				1 % transfer from Q4 to Q5			
	G	G_{GL}	G_{AL}	G_{CT}	G	G_{GL}	G_{AL}	G_{CT}
Australia	1.267	0.343	1.088	2.046	1.267	0.183	0.656	1.309
Belgium	1.712	0.314	1.326	2.594	1.712	0.202	0.857	2.294
Brazil	0.739	0.498	1.003	1.671	0.739	0.183	0.500	0.631
India	1.451	0.341	1.254	2.606	1.451	0.207	0.802	2.392
Sieraleone	0.684	0.459	1.038	0.750	0.684	0.114	0.356	0.122
Slovak Republic	2.203	0.312	1.682	3.819	2.203	0.221	1.134	4.166
USA	1.066	0.336	0.950	1.344	1.066	0.167	0.539	0.781

* Different column shows how the values of the indices change in response to 1 per cent transfer of resources between two consecutive groups.

Q: Quintile, G: Gini coefficient, G_{GL}: Logarithmic transformation of the geometric index, G_{Al}: Logarithmic transformation of the second area measure, G_{CT}: The trigonometric measure based on cotangent of left-hand side complementary angle of all triangles below the Lorenz curve.

Table4. Sensitivity of different measures of inequality*

Country	1 % transfer from Q5 to Q4				1 % transfer from Q4 to Q3			
	G	G_{GL}	G_{AL}	G_{CT}	G	G_{GL}	G_{AL}	G_{CT}
Australia	-1.267	-0.175	-0.645	-1.126	-1.267	-0.305	-1.058	-1.463
Belgium	-1.712	-0.193	-0.844	-1.871	-1.712	-0.279	-1.295	-1.632
Brazil	-0.739	-0.173	-0.486	-0.557	-0.739	-0.424	-0.949	-1.180
India	-1.451	-0.198	-0.789	-2.048	-1.451	-0.301	-1.222	-1.677
Sieraleone	-0.684	-0.109	-0.347	-0.110	-0.684	-0.399	-0.960	-0.575
Slovak Republic	-2.203	-0.211	-1.118	-3.262	-2.203	-0.275	-1.646	-2.049
USA	-1.066	-0.160	-0.529	-0.684	-1.066	-0.299	-0.920	-0.988

* Different column shows how the values of the indices change in response to 1 per cent transfer of resources between two consecutive groups.

Q: Quintile, G: Gini coefficient, G_{GL}: Logarithmic transformation of the geometric index, G_{Al}: Logarithmic transformation of the second area measure, G_{CT}: The trigonometric measure based on cotangent of left-hand side complementary angle of all triangles below the Lorenz curve.

Table5. Sensitivity of different measures of inequality*

Country	1 % transfer from Q3 to Q2				1 % transfer from Q2 to Q1			
	G	G_{GL}	G_{AL}	G_{CT}	G	G_{GL}	G_{AL}	G_{CT}
Australia	-1.267	-0.467	-1.951	-2.292	-1.267	-1.157	-5.130	-10.288
Belgium	-1.712	-0.395	-2.264	-2.506	-1.712	-0.851	-5.573	-9.866
Brazil	-0.739	-0.858	-2.031	-2.976	-0.739	-2.420	-5.938	-15.437
India	-1.451	-0.438	-2.126	-2.591	-1.451	-0.758	-4.979	-6.369
Sieraleone	-0.684	-2.644	-3.595	-11.756	0.068♣ Ø	-1.339	-8.315	5.087 Ø
Slovak Republic	-2.203	-0.358	-2.743	-2.842	-2.203	-0.606	-6.241	-8.165
USA	-1.066	-0.553	-1.853	-2.277	-1.066	-1.755	-5.538	-14.326

* Different column shows how the values of the indices change in response to 1 per cent transfer of resources between two consecutive groups.
♣ Exceptional case where % change ≠ constant (as after transfer the condition: $y_1 \le y_2 \le \ldots \le y_n$ is violated and it can be checked from the distribution in table 1). Ø Inequality increased even after reorganisation in the reverse direction. Q: Quintile, G: Gini coefficient, G_{GL}: Logarithmic transformation of the geometric index, G_{Al}: Logarithmic transformation of the second area measure, G_{CT}: The trigonometric measure based on cotangent of left-hand side complementary angle of all triangles below the Lorenz curve.

Table6. Sensitivity of different measures of inequality*

Country	1 % transfer from Q1 to Q5				1 % transfer from Q5 to Q1			
	G	G_{GL}	G_{AL}	G_{CT}	G	G_{GL}	G_{AL}	G_{CT}
Australia	5.070	2.570	9.721	22.674	-5.070	-2.226	-8.783	-16.835
Belgium	6.849	2.018	10.735	22.875	-6.849	-1.816	-9.975	-18.114
Brazil	2.954	6.444	12.812	53.444	-2.954	-4.245	-9.403	-22.875
India	5.806	2.022	9.833	18.935	-5.806	-1.813	-9.116	-14.987
Sieraleone	2.734	25.334	37.235	689.157	-2.666♣	-6.832	-13.216	-32.799
Slovak Republic	8.811	1.680	12.449	24.542	-8.811	-1.544	-11.748	-19.945
USA	4.264	3.591	10.304	30.067	-4.264	-2.909	-8.840	-19.596

* Different column shows how the values of the indices change in response to 1 per cent transfer of resources between two extreme groups.
♣ Exceptional case where % change ≠ constant (as after transfer the condition: $y_1 \le y_2 \le \ldots \le y_n$ is violated here and it can be checked from the distribution in table 1).
Q: Quintile, G: Gini coefficient, G_{GL}: Logarithmic transformation of the geometric index,
G_{Al}: Logarithmic transformation of the second area measure, G_{CT}: The trigonometric measure based on cotangent of left-hand side complementary angle of all triangles below the Lorenz curve.

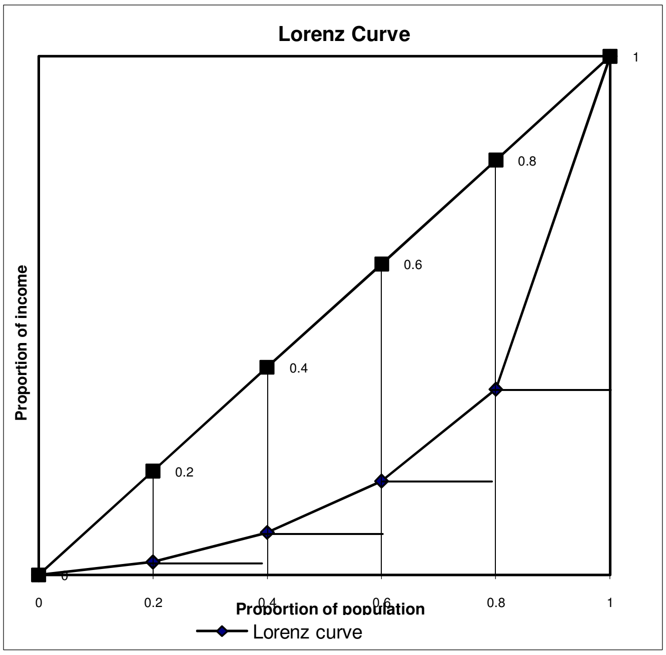

Figure 1. Lorenz curve