A New Pro Transfer-Sensitive Measure of Economic Inequality under the Lorenz Curve Framework in Analogue to the Index of Refraction of Geometrical Optics

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Paper prepared for the 34th IARIW General Conference

Dresden, Germany, August 21-27, 2016

PS1.2: Earnings and Inequality

Time: Monday, August 22, 2016 [Late Afternoon]

A new pro transfer-sensitive measure of economic inequality under the Lorenz curve framework in analogue to the index of refraction of geometrical optics*

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Abstract: Index of refraction is found to be a good measure of economic inequality within the Lorenz curve framework. It has origin in geometrical optics, where it measures bending of a ray of light passing from one homogeneous transparent medium into another. As light refracts according to characteristics of different media, so also Lorenz curve does according to concentration of wealth or income in different strata. With the sole objective of applying this analogy to the Lorenz curve framework, first, I compute refractive (inequality) index for each stratum in a distribution to study condition in each with respect to the ideal condition, and then simply add all and standardise to propose an overall measure for the whole framework. I utilise data on decile group shares of income or consumption for 149 countries from the UNU-WIDER World Income Inequality Database (WIID3.0b), September 2014. Results are lively and remarkable. While a refractive index value of less than 1.00, in case of light, refers an 'anomalous refraction', such a condition of economic inequality is found too common for many of us (50-80 %) in reality. In contrast to that, in most of the countries, the index value of the richest group lies in between the proximities of 2.00 and 5.00, where the same of 1.00 depicts an ideal condition that is enviable. The summative overall measure appears to be pro transfer-sensitive and equivalent to those based on the length of the Lorenz curve and consequently goes beyond the Gini coefficient, which is simply transfer-neutral.

Keywords: Anomalous inequality, Geometrical optics, Gini coefficient, Refractive inequality index, Refractive Lorenz index

JEL classification: D310, D630, O150.

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* Paper accepted for presentation at the 34th IARIW (International Association for Research in Income and Wealth) General Conference in Dresden, Germany, 21-27 August 2016. Different preliminary versions of it appeared as Majumder (2014) and Majumder (2015). I am grateful to Francesco Andreoli, Kaushik Basu, Patrick Moyes, S. Subramanian, and V. Upadhyay for comments and suggestions. I owe my sincere gratitude to Jeremy B. Tatum for enlightening me on advanced technical issues with refractive index, as in physical acoustics. I am also thankful to IARIW, Canada & the University Grants Commission (UGC), India for financial assistance in favour of this submission. Responsibility of error rests with me.

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1. Introduction

Index of refraction is found to be a good measure of economic inequality within the Lorenz curve framework. It has origin in geometrical optics, which deals with the propagation of light by geometrical means and establishes some fundamental principles on refraction of light and the law by which it is governed, such as Snell's law etc. (Mazumdar 1983, pp. 1-4). Whenever a ray of light proceeds from one homogeneous transparent medium into another, its path is bent at the junction of these two media and this bending of ray is called refraction of light. Index of refraction or refractive index is a quantity, which measures the extent of bending of a ray of light in the aforesaid conditions (Jenkins and White 1981, pp. 9-13; Mazumdar 1983, pp. 1-4). Such a concept is akin to that of the Gini coefficient under the Lorenz curve framework, as the latter measures the extent to which the distribution of income or consumption expenditure among individuals or groups within an economy deviates from a perfectly equal distribution. If we consider the unit square of the Lorenz curve framework superimposing the ideas of geometrical optics on it, we realise that in case of an ideal condition, light (or equivalently the Lorenz curve) passes diagonally without refraction. In the presence of inequality, however, it deviates from the hypothetical line of absolute equality and is seen to refract while passing from one stratum into another. The sole objective of this paper is to apply this analogy to the Lorenz curve framework and study the inequality conditions across income groups and distributions. Consequently, I use simple mathematical tools (following Snell's law) to compute refractive inequality index (say, RII) for each stratum or income group as a measure of inequality associated with it with respect to the ideal condition, and treat a simple summation of those (after standardisation) for all the strata as an overall measure of inequality for the whole Lorenz curve framework (say, refractive Lorenz index – RLI). The exercise is done utilising data on decile group shares of income or consumption from the UNU-WIDER World Income Inequality Database (WIID3.0b), September 2014 (UNU WIDER 2014). Data mining is done for 149 countries (as per availability of required information) for different time points leading to 2587 cases stretching from 1936 to 2012, which are again classified according to seven regions, namely Africa, Americas, Asia, Europe, Middle East, Oceania, and Post-Soviet. In this context, it is to be mentioned that although the RII and the RLI are computed for ten income groups, the exercise can be extended vividly to the cases when number of groups or individuals is sufficiently large or when the Lorenz curve is continuous.

Although literature on alternative and intuitively simpler derivations of Gini coefficient has grown exponentially over the years, any previous attempt (other than by this author) to assimilate the idea of refraction of light or sound with that of economic inequality based on Lorenz curve framework is not known. Popular survey papers by Xu (2004) and Yitzhaki and Schechtman (2013, pp. 11-31) do not reveal presence of any study on the approach under discussion. However, it is observed that after aggregation of the refractive indices for all the strata, the overall index (RLI) becomes equivalent to a standardised measure that can be expressed as a ratio of the length of the deviated Lorenz curve to that in the ideal condition, as proposed by Amato (1968, p. 261) and Kakwani (1980, pp. 83-85). This linkage between the measures based on the index of refraction and the length of the Lorenz curve puts the present research in advantageous position. Kakwani (1980, pp. 83-85) discussed about transfer-sensitivity property and proved that unlike the Gini coefficient, the measure based on the length of the Lorenz curve is more sensitive to transfers at the lower levels of income, making it particularly applicable to problems such as measuring the intensity of poverty. Subramanian (2010, 2015) made it clear that the transfer-neutral Gini coefficient is a linear convex combination of two measures which are anti transfer-sensitive and pro transfer-sensitive respectively. According to him, the pro transfer-sensitivity of the latter is reminiscent of a similarly ‘left-wing’ inequality measure derived from the Lorenz curve, which is based on the length (rather than area, as in the case of the Gini coefficient) of the Lorenz curve, as advanced by Amato (1968, p. 261), Kakwani, (1980, pp. 83-85) and the one based on index of refraction as proposed by this author in Majumder (2014)[1]. Further, the proposed measure has several advantages in its practical application, as it is: (i) applicable in part (for different segments of a distribution, as RII) and as a whole (for the complete Lorenz curve framework, as RLI), (ii) additive, and (iii) interpretable as per the scientific propositions of both economics and geometrical optics[2].

The workability of the new proposed measure, as mentioned above, addresses the issue raised by Piketty (2014, p. 266). He preferred to study inequality conditions at different levels of an income distribution separately instead of using a single summary measure, such as Gini coefficient, as the social reality and economic and political significance of inequality are very different at different levels of a distribution.

The paper is organised as follows. Section 2 and 3 describe computation procedures in discrete and continuous cases respectively. Section 4 presents results on RII in some countries and regions. Section 5 is devoted on results on RLI in some countries and regions. Section 6 explores the relationship between RLI and Gini coefficient. Section 7 describes properties of the RLI. Section 8 presents conclusion followed by references.

2. Computation procedures: discrete case

2.1. Refractive inequality index (RII)

In optics, Snell's law of refraction (see Elert 2015, and Jenkins and White 1981, pp. 9-13) exhibits the relationship between different angles of light as it passes from one transparent medium into another as follows:

$$r_a . \sin(\theta_a) = r_w . \sin(\theta_w), \tag{1}$$

where r_a is the refractive index of the medium a the light is leaving, θ_a is the angle of

[1] It is to be mentioned that Subramanian (2015) is in response to Majumder (2014).
[2] And / or Physical Acoustics, as mentioned in footnote 5.

incidence, r_w is the refractive index of the medium w the light is entering, and θ_w is the angle of refraction. An illustration of refraction (from air to water) is shown in figure 1.

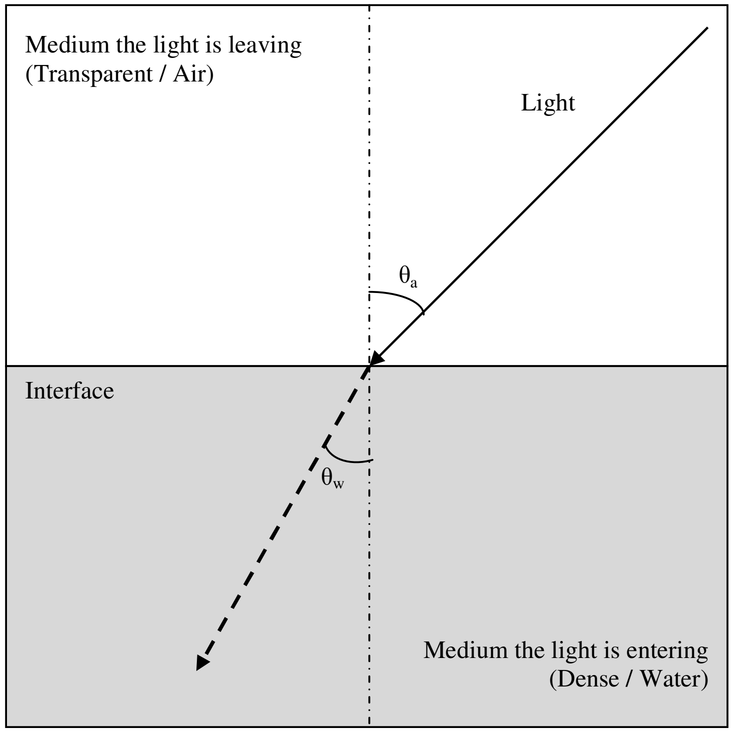

Figure 1. An illustration of refraction (with vertical normal)

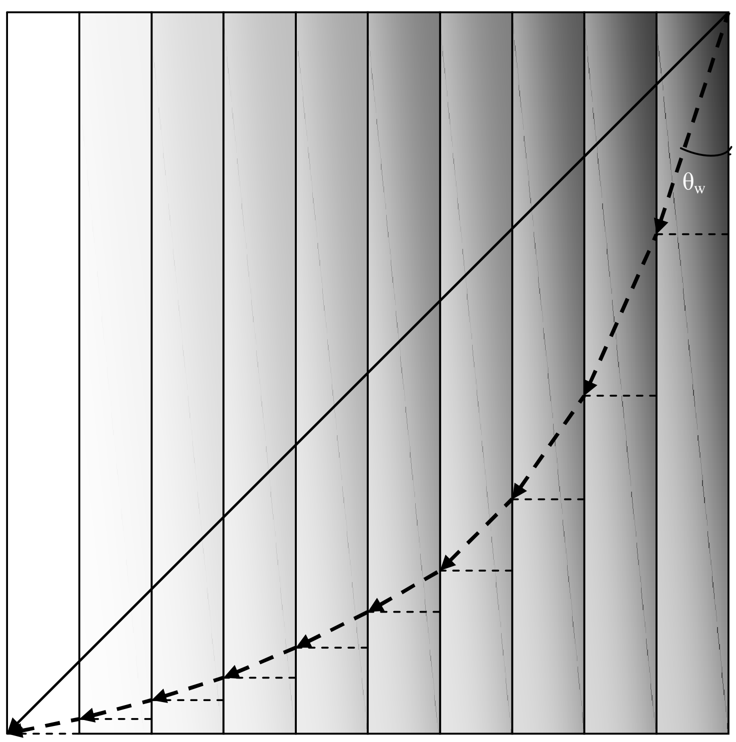

Figure 2. Lorenz curve framework with ten income groups

We may apply formula (1) to the Lorenz curve framework as demonstrated in figure 2 (with standard concept and notations), where we have ten different strata with p_i as proportion of population and y_i as the proportion of income or consumption such that $\sum y_i = 1$ (for i = 1, 2, .., 10 or 1, 2, …, n in general). In that, an ideal condition is the one where light passes diagonally without refraction. As inequality exists, light refracts ten times (as we have considered ten different strata) while passing from one stratum into another.

From figure 2 we may check that there are 10 different triangles associated with ten different strata. Hypotenuses of all the triangles constitute the Lorenz curve. If we assume that light passes from the upward direction (from right to left), the perpendicular of a triangle is 0.10 (i.e., 1/n = proportion of population, p_i) and the base is y_i. The hypotenuse of each triangle (say, h) is:

$$\sqrt{(p_i)^2 + (y_i)^2} \text{ , and} \tag{2}$$

$$\sin(\theta_w) = \frac{p_i}{\sqrt{(p_i)^2 + (y_i)^2}} . \tag{3}$$

The refractive index of the stratum where light enters may be computed with respect to that of the immediate preceding one or relative to that of the ideal condition, where $\theta = 45^0$ with respect to the vertical normal. As the latter seems simple, we compute the index of refraction following the latter. The index of refraction of a particular stratum is [from equation (1)]:

$$r_w = r_a \cdot \frac{\sin(\theta_a)}{\sin(\theta_w)} . \tag{4}$$

As in case of a fully transparent medium and / or in ideal condition the refractive index is 1.00 (by assumption) and the angle of incidence (θ_a) is 45^0,

$$RII = 1 . \frac{\sin(45^0)}{\dfrac{p_i}{\sqrt{(p_i)^2 + (y_i)^2}}} . \tag{5}$$

$$= \frac{\sin(45^0)\sqrt{(p_i)^2 + (y_i)^2}}{p_i}, \tag{6a}$$

$$= n \,.\, \sin(45^0)\,.\,h\,, \tag{6b}$$

as $p_i = 1/n$. RII = refractive inequality index, and h = hypotenuse of each triangle under the Lorenz curve (or part of the Lorenz curve in a stratum) as mentioned in expression (2).

Expression (6b) may also be presented as a ratio of the part-length of the deviated Lorenz curve within a stratum (i.e., truncated Lorenz curve in a stratum) to the length of the Lorenz curve in ideal the condition. As sin (45^0) = 0.71 or $1/\sqrt{2}$, and as $\sqrt{2}$ = length of the Lorenz curve in the ideal condition (say, v),

$$\text{RII} = \frac{n}{\sqrt{2}}.h\,, \tag{6c}$$

$$= n.\frac{h}{v}\,, \tag{6d}$$

$$= n.\frac{\text{part length of the deviated Lorenz curve within a stratum}}{\text{length of the Lorenz curve in the ideal condition}}\,. \tag{6e}$$

Refractive inequality index for each stratum can be obtained easily from expression (6a) for particular values of p_i and y_i. When y = 0, RII (minimum) = sin (45^0) = 0.71; when y = p (everybody has equal share of income), RII (ideal) = sin (45^0)*$\sqrt{2}$ = 1.00; when y = 1.00 (one individual or group assumes all income), maximum value of RII depends upon p (or n). For example, when p = 0.10 (or n = 10) and y = 1.00, RII (maximum) = 7.11.

In general, the maximum value of RII (in the extreme case) can be derived from the following expression:

$$\text{RII}_{max} = \sqrt{(1+n^2)/2}\,. \tag{7}$$

2.2. Refractive Lorenz index (RLI)

If we add all the RIIs, as in expression (6d) for all the strata (for i = 1, 2, …, n) we get:

$$L = n.\frac{u}{v}, \tag{8a}$$

where, L = the overall measure of inequality (before standardisation), and u = $\sum h$ = length of the deviated Lorenz curve. From expression (8a) it appears that the overall measure of inequality is nothing but the ratio of the full-length of the deviated Lorenz curve to the length of the Lorenz curve in the ideal condition as shown below.

$$. \quad L = n.\frac{\text{full length of the deviated Lorenz curve}}{\text{length of the Lorenz curve in the ideal condition}}. \tag{8b}$$

In the extreme case, for n=10, when all resources are given to one group or individual, (in figure 2) the u takes an upward turn from point (0, 0.9). So, the length of the maximum inequality Lorenz curve is (for n = 10)[3]: $0.9 + \sqrt{(0.10)^2 + (1)^2} = 1.905$. In the ideal case, v = u. So, for n=10, from equation (8a),

$$L_{min} = 10.00. \tag{9}$$

In the extreme case (for n = 10), from equation (8a),

$$L_{max} = 10.\frac{1.905}{\sqrt{2}} = 13.47. \tag{10}$$

In general, in the extreme case,

$$L_{max} = \frac{1}{\sqrt{2}}\{(n-1) + \sqrt{1+n^2}\}. \tag{11}$$

If we want results in a normalised 0-100 scale, the refractive Lorenz index (RLI) may be defined as:

$$RLI = 100.\frac{L - L_{min}}{L_{max} - L_{min}}. \tag{12}$$

One may check that expression (8a) or (8b) or (12) is equivalent to the measures proposed by Amato (1968, p. 261) and Kakwani (1980, pp. 83-85).

[3] The maximum length is 2 when n is sufficiently large.

3. Computation procedures: continuous case

Snell's law of the form 'r sin (θ) = constant', as demonstrated above, is useful in studies when a ray of light passes through different media with refractive index being piece-wise constant for each of the medium. In continuous case, there are infinite numbers of infinitesimally narrow groups or strata with continuously varying refractive index throughout the unit square. In such a case, the refractive index is to be computed using a differential form of Snell's law (simply by differentiation of the above expression), as shown below.

$$r.\sin(\theta) = \text{const.} \tag{13}$$

Differentiating the above,

$$r.\cos(\theta) + \sin(\theta).\frac{dr}{d\theta} = 0, \tag{14}$$

or, $$\frac{\cos(\theta)}{\sin(\theta)} = -\frac{1}{r}.\frac{dr}{d\theta}, \tag{15}$$

or, $$\cot(\theta)\,d\theta = -\frac{dr}{r}. \tag{16}$$

Expression (16) shows the differential form of Snell's law when refraction is considered with respect to the vertical normal (Arovas 2008, pp. 2-3 and Tatum 2014, p. 31).

Before proceeding further, the angular description is changed to reap some mathematical advantages[4], as shown in figure 3. It illustrates the case of refraction with respect to horizontal normal where, as per sign convention the angles are of opposite signs. With these, the Snell's law takes the following form (Tatum 1999; Blackstock 2000, pp. 284-285)[5, 6]:

$$r.\cos(\theta) = \text{const.} \tag{17}$$

Differentiating the expression (17),

[4] To express the refractive index in terms of the slope of the tangent line to Lorenz curve.
[5] Both the authors derived differential form of Snell's law in the field of Physical Acoustics, where acoustic weave or ray of sound obeys Snell's law as in case of Optics.
[6] One should take care that figures 5.1 and 5.2 in Arovas (2008) correspond to equation (17) and the derivation presented by him corresponds to the equation (13) as shown above (in the present paper).

$$\tan(\theta)\,d\theta = \frac{dr}{r} \ . \tag{18}$$

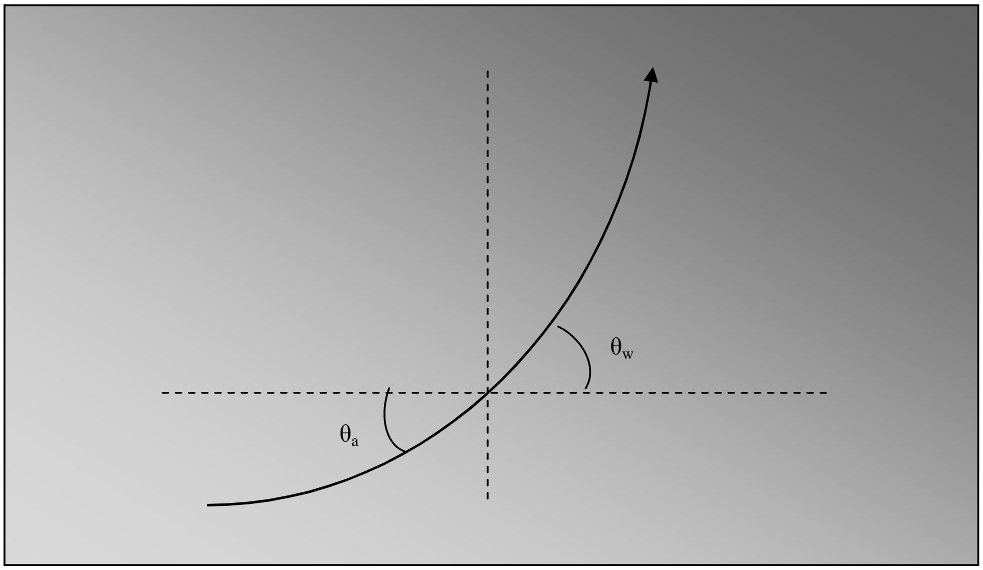

Figure 3. An illustration of refraction in continuous case (with horizontal normal)

As i and θ are continuous functions of the coordinate x, expression (18) may be rewritten as follows:

$$\tan(\theta).\frac{d\theta}{dx} = \frac{1}{r}\frac{dr}{dx} \ . \tag{19}$$

If we express the path as y = y (x),

$$\tan(\theta) = y' \text{ , and} \tag{20}$$

$$\theta' = \frac{d}{dx}\tan^{-1} y' , \tag{21}$$

$$= \frac{y''}{1+y'^2} \ . \tag{22}$$

Replacing the results of (20) and (22) in (19), we have:

$$y'.\frac{y''}{1+y'^2} = \frac{r'}{r} \ , \tag{23}$$

$$\text{or, } \ r' = y'.\frac{y''}{1+y'^2}.r \ . \tag{24}$$

As the quantities in the right-hand side (with the first-order derivative being the slope of the tangent line to the Lorenz curve and r being the initial refractive index) are known, r' or change in the refractive index due to the tiniest change in proportion of population (measured along x axis) can be known.

In continuous case, the refractive Lorenz index (RLI), which is based on the length of the Lorenz curve, can be computed simply by replacing the summation used in case of equation (8a) by an integral.

Further, in continuous case, there is a point on the Lorenz curve where the slope of the tangent line is equal to that of the diagonal one. This is the point of inflection, as it divides the population into two groups with an RII value of < 1.00 in the left and > 1.00 in the right. This concept may be used to derive a line of inequality in accordance with that of poverty.

4. Results on refractive inequality index in some countries and regions

Refractive inequality index (RII) is computed following formula (6a). Results of some countries (selected arbitrarily) in seven regions are displayed in table 1 below[7].

Interpretation of results is simple. In the ideal condition, RII = 1.00 [as discussed in relation to expressions (6a) to (6e)]. An index value of 1.00 is desirable for each of the strata. Deviation from 1.00 is undesirable. Any value of less than 1.00 is strictly undesirable. Standard literature in optics maintains that an index value of less than 1.00 (in case of light) does not represent a physically possible system (Nave 2012)[8]. Further, in case of light, a refractive index value of less than 1.00 represents an 'anomalous refraction' (Feynman 2011, p. 33-9). However, the condition, which does not represent a physically possible system or which is considered 'anomalous' in physical science, appears to be too common for many of us (50-80 %) in reality. For example, in table 1, we see that 80 % common mass in South

[7] A more detailed table is also available in the Annexure I.

[8] Except some very special cases, where refractive index is lower than but very close to 1.00. It occurs with the refraction of x-rays, and also with visible light in the immediate vicinity of a spectrum line. However, specialist literature on this issue rests beyond common understanding, as I realised it thanks to my conversation with Jeremy B. Tatum.

Africa, both in 1997 and 2008, is subject to such a condition of ‘anomalous inequality’ (i.e., RII < 1.00 for the first eight consecutive income groups). After analysing 2587 cases, it has been found that percentage of people under the condition of ‘anomalous inequality’ varies from 50 to 80. There are 20 countries (18 European countries with Cuba and Yemen in different years leading to 77 cases), where concentration of people under the condition ‘anomalous inequality’ is the lowest (50 %). In table 1, France (in 2001) and Yemen (in 1998) are seen to experience the same condition. On the contrary, there are 35 countries [16 from Africa, 13 from Latin America, five from Asia and one from Europe (Germany in 1955, 1950 and 1964) leading to 83 cases in different years], where concentration of people under the anomalous condition is the highest (80 %). In table 1, South Africa (in 1997 and 2008) and Zambia (in 1991) and Pakistan (in 1996) are seen to experience the same condition.

RII with a value of more than 1.00 indicates higher concentration of wealth or income with respect to the ideal condition [as discussed in case of expression (6a)]. Although hypothetically, in case of ten income groups, RII ranges from 0.71 to 7.11, an analysis of 2587 cases stretching from 1936 to 2012, reveals that RII, for the richest group, reaches to 5.02 (Zambia in 1991) as shown in table 1. An RII value of 5.02 indicates significantly higher concentration of wealth or income in one group in contrast to the ideal condition as well as the condition ‘anomalous inequality’ of the majority within the income distribution.

In continuation with the above, it is further observed that when RII exceeds 2.50 (for the richest income group), 70 % common mass lives under the condition of ‘anomalous inequality’. When RII exceeds 3.63, the said percentage figure rises to 80. Many African countries with Latin American ones are seen to experience such conditions. Yemen (in 1998), with an RII value of 1.36 for the richest income group, remains at the bottom of the list with the least percentage of common mass under the anomalous condition of inequality.

Table 1. Refractive Inequality Index (RII) and Refractive Lorenz Index (RLI) in some selected countries

Region	Country	Year	Gini	RII_1	RII_2	RII_3	RII_4	RII_5	RII_6	RII_7	RII_8	RII_9	RII_{10}	RLI
Africa	South Africa	1997	54.5	0.71	0.73	0.74	0.75	0.77	0.79	0.83	0.90	1.09	3.90	35.4
	South Africa	2008	59.4	0.71	0.71	0.72	0.72	0.73	0.75	0.80	0.93	1.39	4.14	46.2
	Zambia	1991	77.3	0.71	0.71	0.71	0.72	0.73	0.74	0.77	0.83	1.03	5.02	56.5
	Zambia	2004	50.0	0.71	0.71	0.72	0.75	0.77	0.88	0.97	1.33	1.67	2.62	32.5
Americas	Brazil	1999	57.0	0.71	0.72	0.73	0.75	0.78	0.82	0.89	1.02	1.35	3.29	31.0
	Brazil	2009	52.0	0.71	0.73	0.75	0.77	0.81	0.85	0.92	1.04	1.31	3.01	26.1
	Canada	1997	31.7	0.73	0.78	0.82	0.86	0.91	0.96	1.03	1.13	1.30	1.83	10.4
	Canada	2007	31.5	0.73	0.78	0.81	0.85	0.89	0.95	1.02	1.12	1.28	1.96	11.5
	United States	2000	39.4	0.72	0.75	0.78	0.83	0.88	0.94	1.01	1.13	1.34	2.17	15.7
	United States	2010	37.3	0.71	0.75	0.78	0.82	0.87	0.93	1.02	1.14	1.36	2.21	16.9
Asia	India	1999	31.7	0.74	0.77	0.80	0.83	0.87	0.91	0.98	1.09	1.30	2.16	13.2
	India	2005	48.0	0.71	0.72	0.74	0.76	0.79	0.83	0.91	1.06	1.42	3.04	28.6
	Pakistan	1970	14.6	0.86	0.89	0.91	0.93	0.95	0.98	1.01	1.05	1.12	1.39	2.4
	Pakistan	1996	30.6	0.75	0.78	0.80	0.82	0.84	0.87	0.91	0.98	1.10	2.73	17.0
Europe	France	2001	27.0	0.76	0.79	0.86	0.86	0.90	1.00	1.05	1.10	1.27	1.69	7.8
	France	2011	30.8	0.75	0.79	0.83	0.86	0.90	0.94	1.00	1.08	1.22	1.96	9.9
	Germany	2001	24.0	0.76	0.82	0.86	0.90	0.94	0.99	1.03	1.11	1.22	1.57	6.1
	Germany	2011	29.0	0.74	0.80	0.84	0.88	0.92	0.97	1.04	1.12	1.25	1.77	8.8
Middle East	Israel	1997	35.8	0.73	0.76	0.79	0.83	0.88	0.95	1.02	1.14	1.33	2.01	13.0
	Israel	2007	36.9	0.72	0.74	0.77	0.81	0.86	0.93	1.02	1.15	1.37	2.20	17.0
	Yemen	1992	21.8	0.73	0.75	0.79	0.82	0.87	0.92	0.99	1.09	1.29	2.29	15.6
	Yemen	1998	39.5	0.76	0.81	0.86	0.91	0.97	1.02	1.08	1.16	1.24	1.36	5.3
Oceania	Australia	1989	33.3	0.73	0.78	0.81	0.85	0.89	0.95	1.03	1.14	1.33	1.90	11.5
	Australia	2003	31.2	0.73	0.78	0.81	0.84	0.89	0.95	1.02	1.14	1.32	1.91	11.5
Post-Soviet	Armenia	2003	48.4	0.71	0.73	0.76	0.79	0.83	0.88	0.95	1.09	1.31	2.71	22.2
	Armenia	2011	-	0.74	0.78	0.81	0.84	0.88	0.93	0.99	1.08	1.25	2.10	11.8
	Russian Federation	1988	23.8	0.77	0.82	0.86	0.90	0.94	0.99	1.05	1.12	1.23	1.55	6.0
	Russian Federation	1998	48.7	0.72	0.73	0.76	0.79	0.82	0.88	0.95	1.06	1.27	2.83	22.9

RII: Refractive Inequality Index (subscripts denote income groups or strata from the lower end), RLI: Refractive Lorenz Index
Source: Gini coefficient - WIID3.0b), September 2014; Self-elaboration, otherwise

Table 2. Refractive Inequality Index (RII) and Refractive Lorenz Index (RLI) in the regions (for initial and final years in data set)*

Region	Period	No. of countries	RII_1	RII_2	RII_3	RII_4	RII_5	RII_6	RII_7	RII_8	RII_9	RII_{10}	RLI
Africa	Old days	29	0.72	0.74	0.76	0.78	0.81	0.86	0.93	1.05	1.29	2.91	24.99
Africa	Recent days	29	0.72	0.73	0.75	0.77	0.80	0.85	0.93	1.05	1.30	3.05	27.67
Americas	Old days	27	0.72	0.74	0.76	0.79	0.83	0.88	0.96	1.08	1.32	2.67	21.82
Americas	Recent days	27	0.72	0.74	0.76	0.79	0.83	0.89	0.96	1.09	1.32	2.66	21.91
Asia	Old days	19	0.74	0.77	0.80	0.82	0.86	0.91	0.97	1.08	1.27	2.33	15.88
Asia	Recent days	19	0.73	0.76	0.79	0.82	0.86	0.91	0.98	1.09	1.29	2.36	16.75
Europe	Old days	30	0.74	0.78	0.82	0.86	0.91	0.96	1.03	1.12	1.27	1.90	11.30
Europe	Recent days	30	0.74	0.80	0.84	0.88	0.92	0.97	1.03	1.12	1.25	1.76	8.84
Middle East	Old days	7	0.73	0.75	0.78	0.81	0.85	0.90	0.97	1.09	1.31	2.42	17.92
Middle East	Recent days	7	0.73	0.76	0.79	0.82	0.86	0.91	0.99	1.09	1.31	2.32	16.94
Oceania	Old days	3	0.73	0.77	0.80	0.84	0.88	0.95	1.02	1.14	1.33	1.99	13.06
Oceania	Recent days	3	0.72	0.76	0.79	0.82	0.86	0.92	1.00	1.12	1.32	2.23	15.72
Post-Soviet	Old days	14	0.76	0.80	0.84	0.87	0.92	0.97	1.03	1.11	1.25	1.76	8.85
Post-Soviet	Recent days	14	0.73	0.76	0.79	0.83	0.88	0.93	1.01	1.12	1.32	2.14	14.99
All	Old days	129	0.74	0.76	0.79	0.82	0.86	0.91	0.98	1.09	1.29	2.36	17.39
All	Recent days	129	0.73	0.76	0.79	0.82	0.86	0.91	0.98	1.09	1.30	2.41	18.24

RII: Refractive Inequality Index (subscripts denote income groups or strata from the lower end), RLI: Refractive Lorenz Index

* The full table is shown in the Annexure I.

Source: Self-elaboration

In order to have a region-wise picture, I select 129 countries for two time points such that the results are somewhat comparable over time. However, it is to be mentioned that time points are not fixed for all the countries. A country is chosen, as per availability of data, for the initial and final years in the data set and those are termed as ‘old days’ and ‘recent days’ as shown in table 2. We may see that mean RII values of the richest group in the said seven regions are as follows (in recent days): 3.05 (Africa), 2.66 (Americas), 2.26 (Asia), 1.76 (Europe), 2.32 (Middle East), 2.23 (Oceania), and 2.14 (Post-Soviet). As compared to the results of ‘old days’, all the regions (except in Europe and Middle East) marked in increase in concentration of wealth or income in the highest income groups.

5. Results on refractive Lorenz index in some countries and regions

Refractive Lorenz index (RLI) is computed using formulae (8a) and (12). It is nothing but the summation of all the RIIs of the ten different income groups or strata expressed in a 0-100 point normalised scale. Values of RLI are displayed in the final columns of tables 1 and 2 (and table 5 in Annexure I). Interpretation of the RLI is similar to that of Gini coefficient. Although hypothetically, RLI ranges from 0 to 100, an analysis of 2587 cases stretching from 1936 to 2012, reveals actual minimum and maximum as 2.4 (Pakistan in 1970) to 56.5 (Zambia in 1991) respectively indicating the lowest and highest levels of inequality as per the data set in reality (as shown in table 1).

Table 1 also shows changes in RIIs and RLIs over a period of ten years or so in the countries selected arbitrarily. For example, over a period of ten years in Germany, RLI increased from 6.1 (2001) to 8.8 (2011) indicating an increase in economic inequality in the Country. A close observation will reveal that such an increase in RLI is due to the decrease in RIIs (as undesirable) for the income groups, where those were less than 1.00 simultaneously with the increase in the same (as undesirable) for the income groups where those were more than 1.00. To cite another example, we see that in Armenia over a period of eight years or so, RLI decreased from 22.2 (in 2003) to 11.8 (in 2011) indicating a decrease in economic inequality in the Country. A close observation will reveal that such a decrease in RLI is due to the increase in RIIs (as desirable) for the income groups, where those were less than 1.00 simultaneously with the decrease in the same (as desirable) for the income groups where those were more than 1.00. The spirit of these examples is equally applicable for all the countries. In case of Australia, we see that RLI does not change in between 1989 and 2003. Again, we may check that RIIs (in Australia) for the income groups remain almost constant (indicating almost constant concentration of wealth or income) over the years. Table 2 shows changes in RII and RLI in seven regions.

In order to see about how (empirically) change in one RII (holding others constant) brings change in the RLI, I opt for a multivariate analysis. The exercise is done by estimating Cobb-Douglas type functions, results of which are presented in table 3.

Table 3. The Summary and goodness of fit statistics of the Cobb-Douglas type function

Statistic	Value	Standard error	F or t*	Sig.
R / Adjusted R square	0.999 / 0.998	0.02585	130723.51	0.000
Constant	-0.289	0.008	-37.064	0.000
ln (RII_1)	-3.488	0.060	-58.133	0.000
ln (RII_2)	-2.067	0.100	-20.766	0.000
ln (RII_3)	-1.086	0.105	-10.390	0.000
ln (RII_4)	-0.538	0.085	-6.354	0.000
ln (RII_5)	-0.468	0.072	-6.516	0.000
ln (RII_6)	-0.118	0.072	-1.640	0.101
ln (RII_7)	0.098	0.060	1.635	0.102
ln (RII_8)	0.431	0.047	9.161	0.000
ln (RII_{10})	1.042	0.030	35.029	0.000

Dependent variable: Refractive Lorenz Index (RLI); n = 2587
* F for adjusted R square, t for the constant and the coefficients
ln: Natural logarithm, RII: Refractive Inequality Index (subscripts denote income groups or strata from the lower end), Variable excluded from the models: ln (RII_9)
Source: Self-elaboration

Table 3 shows some important empirical results revealing the essential property of the new proposed measure. As the RLI is additive, one may confirm that each component of it maintains the spirit of the Pigou-Dalton condition. For example, the coefficient of RII_1 is: - 3.488. It implies that when RII of the first income group increases by one per cent (i.e., when concentration of wealth or income increases), RLI decreases by 3.488 per cent (implying a decrease in overall inequality). This negative relationship stands significant for the first six consecutive income groups. We know that in most of the 2587 income distributions, 50 % or more common mass lives under the condition of 'anomalous inequality' (with RII < 1.00). So, when concentration of wealth of income increases in these income groups, overall inequality shows a decline. In general, for the stratum where value of RII is less than 1.00, in response to any inward transfer to it, RLI decreases and vice-versa. On the contrary, we know that for the richer income groups, value of RII is more than 1.00. In such a situation, when it increases further (implying further increase in concentration of wealth or income), RLI increases, as can be checked from table 3. It is prominent from the results that major diminution in overall inequality may come from the positive and negative changes at the lower and upper ends respectively.

6. Relationship between refractive Lorenz index and Gini coefficient

Gini coefficient and RLI are closely related to Lorenz curve. The former is equal to twice the area bounded by the deviated Lorenz curve and that in the ideal condition. The latter is the ratio of the deviated Lorenz curve to that in the ideal condition. An empirical examination reveals that both the measures are perfectly correlated by power equation as shown in table 4 and in figure 4. As, RLI is obtained from the grouped data on distribution of income or consumption, the relationship is explored after computing Gini coefficient from the same data following the standard measure under the mean difference approach[9, 10].

I estimate a model with the 2587 cases as mentioned previously. It is found, that nearly 100 % variability in the RLI is explained by (natural logarithm of) the Gini coefficient with an adjusted R square value of nearly 1.00. This finding supports those of Majumder (2014)

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[9] Say, G_3 in Anand (1983, p. 313) after multiplying it by 100.

[10] When Gini coefficient is computed from grouped data, it assumes lower value than that based on micro data.

and Majumder (2015), which used data on quintile share of income or consumption from the World Development Indicators 2014.

.Table 4. The Summary and goodness of fit statistics of Power model

Statistic	Value	Standard error	F or t*	Sig.
R / Adjusted R square	1.000/0.999	0.158	4538833.247	0.000
Constant	0.015	0.852	308.173	0.000
ln (Gini coefficient)	1.909	0.000	2130.454	0.000

Dependent variable: Refractive Lorenz Index (RLI)
* F for adjusted R square, t for the constant and the coefficients, ln: Natural logarithm
Source: Self-elaboration

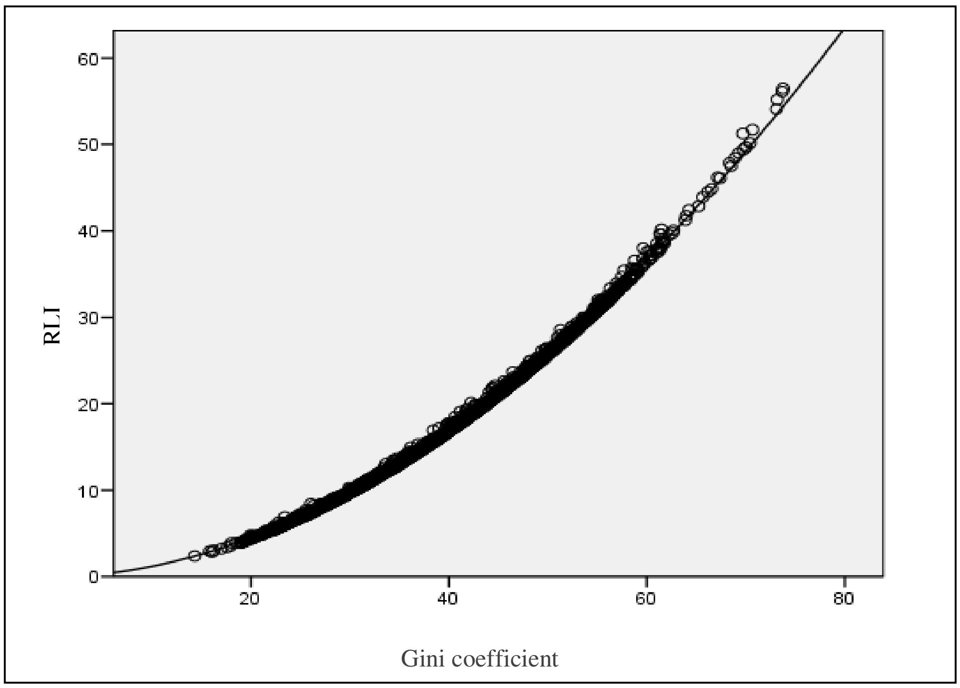

Figure 4. Gini coefficient vs. Refractive Lorenz Index (n =2587)

However, such an empirical relationship holds good when there exists one-to-one correspondence between Gini coefficient (or the bounded area) and the length of the deviated Lorenz curve. For example, if two (or more) different Lorenz curves represent the same bounded area (i.e., Gini coefficient), the said relationship will break theoretically. Such a possibility of having the same Gini coefficient for different Lorenz curves is presented in the next section[11].

[11] One may also relate it with the idea of 'Adanac' as presented by Osberg (1981, p. 14). It considers a simple two class example in which the Gini coefficient is held constant while the size of the rich and poor changes. In

7. Properties of refractive Lorenz index

RLI belongs to the family of 'left-wing' or pro transfer-sensitive inequality measures as discussed by Subramanian (2015). I cite one simple numerical example to clarify the issue of sensitivity of RLI[12]. Consider the following distributions with five income groups: o = (7, 13, 20, 27, 33), p = (10, 10, 20, 27, 33) and q = (7, 13, 20, 30, 30). It can be seen that p has been derived from o by a downward transfer of 3 income units to the lowest 20 % from the second 20 %; and q has been derived from o by an identical transfer of 3 income units to the fourth 20 % from the highest 20 %. One may check that the areas enclosed by the Lorenz curves represented by p and q with the diagonal of the unit square are the same (and hence, Gini coefficients for the two are the same), although p is skewed towards (0,0) - 'bulges at the top'; and q towards (1,1) - 'bulges at the bottom'.

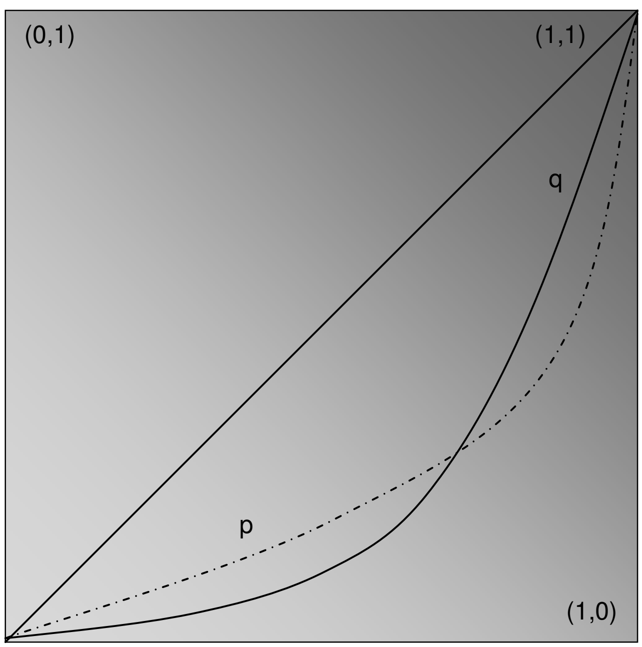

Figure 5. Lorenz curves with different skewness

Figure 5 represents such ideas more clearly. An inequality measure (say, Z), which satisfies the Pigou-Dalton transfer axiom, will be transfer-neutral if Z(o) > Z(p) = Z(q); and Z

such cases, although the bounded area or Gini coefficient remains constant, angles of incidence or the length of Lorenz curves may differ leading to different RIIs and RLIs.

[12] In accordance with Subramanian (2015).

will be pro transfer-sensitive[13] if Z(o) > Z(q) > Z(p). For the numerical example under review, and given equations (8a) and (12) (for RLI, say R) and any standard measure for Gini coefficient (G)[14], it can be verified that G(o) [= 26.4] > G(p) = G(q) [= 25.2]: the Gini coefficient is transfer-neutral; and R(o) [= 10.1] > R(q) [= 9.9] > R(p) [= 9.3]: RLI is pro transfer-sensitive (meaning more sensitive to transfers at the lower end).

RLI is equivalent to the 'New Inequality Measure' of Kakwani (1980, pp. 83-85), which is a strictly convex function of income, which again implies that the measure is sensitive to transfers at all levels of income. Kakwani (1980, pp. 84-85) went further to prove that the measure attaches higher weight to transfers at the lower end than at the middle and upper ends of the distribution, such that weights given to transfers decrease monotonically as income increases[15]. On this point (considering the interests of the poor), with the 'New Inequality Measure' of Kakwani (1980, pp. 83-85), refractive Lorenz index (RLI) too goes beyond the Gini coefficient, which is simply transfer-neutral. Further, the workability of RLI is more appealing thanks to its property of additivity. It is shown that as a summative measure, RLI is applicable in part (as RII) for different segments of a distribution and / or as a whole (as RLI) for the complete one. Also, it is needless to say that the workability of RLI with respect to the property of additivity is far simple than tedious mathematical derivations on the so-called 'decomposition' of Gini coefficient[16].

8. Conclusion

An ideal state of development, when viewed with fantasy, is nothing but a state or condition where light touches everybody without refraction. The diagonal line of the Lorenz curve framework represents such an ideal condition. In the presence of inequality, however, it

[13] The third case is of anti transfer-sensitivity, which requires Z(p) > Z(q) > Z(r); a 'right-wing' inequality measure satisfies this condition.
[14] *Ibid.* 9.
[15] Kakwani (1990, pp. 84-85) proved several Lemmas to describe transfer-sensitive properties of his new inequality measure, which are equally applicable for RLI because of equivalence of it with the former.
[16] Literature on this issue is vast. However, one may refer Anand (1983, pp. 319-326).

deviates or refracts from the ideal condition. Whenever a ray of light proceeds from one homogeneous transparent medium into another, its path is bent at the junction of these two media and this bending of ray is called refraction of light. Index of refraction or refractive index, which has its origin in geometrical optics, measures the extent of bending of a ray of light in the aforesaid conditions. Such a concept is akin to that of the Gini coefficient under the Lorenz curve framework, as the latter measures the extent to which the distribution of income or consumption deviates from a perfectly equal distribution. The sole objective of the paper has been to apply similar analogy to the Lorenz curve framework and propose a new measure of economic inequality, which could be far more functional as compared to the Gini coefficient. Consequently, first, refractive (inequality) index is computed for each stratum in a distribution to study condition in each with respect to the ideal condition, and then all are added simply and standardised to propose an overall measure for the whole framework. The summative overall measure appears to be pro transfer-sensitive (meaning more sensitive to transfers at lower levels of income) and equivalent to those based on the length of the Lorenz curve. The workability of the proposed measure, in parts and as a whole is tested with the UNU-WIDER World Income Inequality Database (WIID3.0b), September 2014 for several countries and found satisfactory. Further, the principles and propositions of economic and physical sciences together make it possible to introduce new vocabulary, such as 'anomalous inequality' as well as distinguish between conditions associated with higher and lower concentration of wealth or income in a group in contrast to the ideal condition. Being overly simple but contented with its properties of additivity and pro transfer-sensitivity, the proposed measure of economic inequality based on the index of refraction of light or sound could be a good substitute of the said transfer-neutral Gini coefficient and similar ones.

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Annexure I

Table 5. Refractive Inequality Index (RII) and Refractive Lorenz Index (RLI) in some selected countries

Country	Year	Gini	RII_1	RII_2	RII_3	RII_4	RII_5	RII_6	RII_7	RII_8	RII_9	RII_{10}	RLI
Algeria	1988	40.10	0.73	0.76	0.79	0.82	0.86	0.90	0.97	1.07	1.25	2.42	16.1
Algeria	1995	35.30	0.74	0.76	0.80	0.84	0.88	0.94	1.01	1.12	1.32	2.03	12.7
Argentina	1953	41.20	0.74	0.77	0.79	0.81	0.84	0.88	0.92	0.99	1.17	2.69	17.5
Argentina	2011	41.00	0.72	0.75	0.78	0.81	0.86	0.92	1.01	1.14	1.37	2.22	16.7
Armenia	1996	48.20	0.72	0.74	0.75	0.78	0.81	0.85	0.93	1.06	1.31	2.89	24.1
Armenia	2011	37.10	0.72	0.75	0.79	0.83	0.88	0.94	1.04	1.15	1.33	2.08	14.8
Australia	1967	31.20	0.72	0.78	0.83	0.88	0.92	0.97	1.05	1.13	1.28	1.82	10.7
Australia	2003	31.20	0.73	0.78	0.81	0.84	0.89	0.95	1.02	1.14	1.32	1.91	11.5
Austria	1983	28.00	0.74	0.78	0.82	0.88	0.94	1.01	1.07	1.17	1.28	1.61	8.6
Austria	2011	26.30	0.75	0.81	0.85	0.89	0.93	0.98	1.03	1.10	1.21	1.70	7.3
Bangladesh	1963	33.00	0.74	0.80	0.80	0.82	0.92	0.92	1.00	1.09	1.27	2.04	11.6
Bangladesh	2010	45.80	0.72	0.74	0.76	0.79	0.82	0.88	0.95	1.08	1.33	2.63	20.6
Barbados	1952	45.50	0.72	0.73	0.74	0.77	0.83	0.87	0.97	1.14	1.39	2.66	23.1
Barbados	2010	47.00	0.71	0.74	0.76	0.78	0.81	0.86	0.92	1.05	1.28	2.91	23.9
Belarus	1988	22.80	0.77	0.83	0.86	0.90	0.94	0.99	1.04	1.11	1.21	1.53	5.5
Belarus	2003	24.90	0.77	0.83	0.86	0.90	0.93	0.97	1.02	1.09	1.22	1.61	6.0
Belgium	1969	32.30	0.74	0.78	0.82	0.86	0.90	0.94	1.01	1.10	1.26	1.96	10.7
Belgium	2011	26.30	0.75	0.81	0.85	0.89	0.93	0.99	1.05	1.12	1.22	1.65	7.2
Belize	1993	56.00	0.71	0.73	0.75	0.77	0.79	0.83	0.88	0.98	1.17	3.41	29.4
Belize	1999	50.00	0.71	0.73	0.76	0.78	0.82	0.87	0.94	1.06	1.29	2.89	24.3
Bolivia	1986	51.60	0.71	0.73	0.74	0.77	0.81	0.86	0.93	1.06	1.35	2.94	25.8
Bolivia	2008	54.00	0.71	0.72	0.74	0.77	0.81	0.86	0.94	1.07	1.32	3.03	27.7
Botswana	1971	57.40	0.71	0.71	0.73	0.75	0.79	0.84	0.94	1.10	1.47	3.06	31.7
Botswana	2003	57.30	0.71	0.72	0.73	0.75	0.78	0.83	0.92	1.09	1.46	3.09	30.8
Brazil	1960	42.30	0.73	0.75	0.77	0.80	0.84	0.89	0.96	1.09	1.32	2.47	17.8
Brazil	2009	52.00	0.71	0.73	0.75	0.77	0.81	0.85	0.92	1.04	1.31	3.01	26.1
Bulgaria	1957	24.60	0.76	0.82	0.86	0.90	0.94	0.99	1.04	1.12	1.23	1.58	6.4
Bulgaria	2011	35.10	0.73	0.76	0.80	0.85	0.90	0.96	1.03	1.13	1.30	1.99	12.6
Burkina Faso	1994	.	0.72	0.74	0.76	0.78	0.81	0.85	0.92	1.04	1.32	2.81	22.2
Burkina Faso	2003	.	0.73	0.75	0.77	0.80	0.83	0.87	0.94	1.04	1.26	2.71	19.9
Cambodia	1994	38.50	0.73	0.75	0.76	0.78	0.81	0.86	0.93	1.05	1.31	2.74	21.1

Cambodia	1999	37.40	0.73	0.75	0.78	0.80	0.82	0.86	0.92	1.03	1.28	2.72	19.9
Canada	1961	32.20	0.73	0.78	0.82	0.86	0.91	0.97	1.05	1.15	1.32	1.80	10.8
Canada	2007	31.50	0.73	0.78	0.81	0.85	0.89	0.95	1.02	1.12	1.28	1.96	11.5
Chile	1968	37.60	0.72	0.76	0.79	0.83	0.88	0.93	1.02	1.13	1.33	2.10	14.3
Chile	2009	51.00	0.72	0.74	0.75	0.78	0.80	0.85	0.91	1.02	1.27	3.04	25.1
China	1995	33.20	0.75	0.79	0.82	0.85	0.89	0.93	0.99	1.08	1.24	2.06	11.2
China	2002	45.30	0.72	0.73	0.75	0.78	0.82	0.89	1.00	1.18	1.45	2.37	20.5
Colombia	1960	59.20	0.72	0.73	0.73	0.74	0.76	0.79	0.86	0.97	1.22	3.63	33.4
Colombia	2010	54.00	0.71	0.73	0.74	0.76	0.79	0.84	0.91	1.03	1.29	3.18	28.3
Costa Rica	1961	47.20	0.72	0.74	0.76	0.79	0.82	0.88	0.95	1.08	1.31	2.71	21.7
Costa Rica	2010	46.00	0.72	0.74	0.76	0.79	0.82	0.88	0.95	1.08	1.37	2.62	21.0
Cote D'Ivoire	1959	45.60	0.73	0.76	0.78	0.80	0.83	0.87	0.92	1.00	1.14	2.86	19.9
Cote D'Ivoire	2008	44.70	0.72	0.74	0.77	0.80	0.84	0.89	0.97	1.08	1.33	2.56	19.8
Croatia	1998	28.40	0.75	0.79	0.83	0.86	0.90	0.96	1.02	1.12	1.28	1.82	9.3
Croatia	2011	31.00	0.73	0.78	0.82	0.87	0.92	0.98	1.05	1.14	1.30	1.77	10.2
Cuba	1953	55.00	0.71	0.72	0.72	0.73	0.77	0.86	1.03	1.21	1.52	2.83	31.6
Cuba	1978	27.00	0.75	0.77	0.80	0.87	0.97	1.02	1.14	1.22	1.29	1.46	8.3
Cyprus	1966	19.30	0.78	0.85	0.88	0.92	0.97	1.00	1.05	1.11	1.19	1.39	4.0
Cyprus	2011	29.10	0.75	0.80	0.83	0.87	0.91	0.96	1.02	1.10	1.25	1.81	8.8
Czech Republic	1989	19.80	0.81	0.85	0.88	0.91	0.94	0.99	1.04	1.11	1.20	1.42	4.0
Czech Republic	2011	25.20	0.76	0.82	0.86	0.89	0.93	0.96	1.02	1.09	1.20	1.69	6.7
Czechoslovakia	1958	27.10	0.74	0.80	0.85	0.89	0.94	0.99	1.06	1.14	1.27	1.61	7.9
Czechoslovakia	1988	20.10	0.80	0.85	0.88	0.91	0.94	0.98	1.03	1.09	1.19	1.49	4.3
Denmark	1953	40.00	0.71	0.74	0.77	0.82	0.90	0.96	1.04	1.18	1.35	2.10	16.3
Denmark	2011	27.80	0.73	0.82	0.86	0.90	0.94	0.99	1.04	1.11	1.22	1.70	8.4
Dominican Republic	1969	45.50	0.72	0.74	0.77	0.79	0.83	0.88	0.96	1.08	1.31	2.61	20.2
Dominican Republic	2010	45.00	0.72	0.74	0.76	0.79	0.83	0.88	0.96	1.09	1.34	2.60	20.4
Ecuador	1968	52.70	0.71	0.73	0.75	0.77	0.80	0.86	0.93	1.05	1.29	3.03	26.3
Ecuador	2010	47.00	0.72	0.74	0.76	0.79	0.83	0.88	0.95	1.07	1.32	2.69	21.5
Egypt	1965	43.40	0.72	0.74	0.77	0.80	0.84	0.91	1.01	1.15	1.41	2.31	18.8
Egypt	1997	53.80	0.71	0.73	0.74	0.77	0.80	0.84	0.90	1.01	1.25	3.25	28.4
El Salvador	1961	46.30	0.71	0.74	0.76	0.79	0.83	0.89	0.98	1.12	1.38	2.53	21.1
El Salvador	2010	43.00	0.71	0.74	0.77	0.81	0.85	0.91	0.99	1.11	1.35	2.41	18.9
Estonia	1988	23.00	0.77	0.82	0.86	0.90	0.95	1.00	1.06	1.13	1.23	1.48	5.6
Estonia	2011	31.90	0.73	0.78	0.82	0.86	0.90	0.96	1.04	1.13	1.31	1.83	10.6
Ethiopia	1981	32.40	0.75	0.79	0.82	0.86	0.89	0.94	0.99	1.07	1.20	2.07	10.8
Ethiopia	1997	45.90	0.72	0.73	0.75	0.78	0.81	0.85	0.91	1.01	1.23	3.09	25.5
Fiji	1968	42.80	0.72	0.74	0.76	0.80	0.84	0.91	1.01	1.15	1.44	2.26	18.4

Fiji	1991	46.00	0.72	0.74	0.77	0.81	0.84	0.89	0.96	1.08	1.28	2.57	19.4
Finland	1952	41.00	0.71	0.74	0.78	0.82	0.87	0.95	1.04	1.16	1.36	2.16	17.1
Finland	2011	25.80	0.76	0.81	0.85	0.89	0.93	0.98	1.03	1.11	1.22	1.67	7.0
France	1956	48.00	0.71	0.72	0.74	0.80	0.86	0.90	1.02	1.13	1.39	2.51	22.5
France	2011	30.80	0.75	0.79	0.83	0.86	0.90	0.94	1.00	1.08	1.22	1.96	9.9
Gabon	1960	69.00	0.71	0.71	0.72	0.73	0.75	0.78	0.83	0.93	1.15	4.16	42.4
Gabon	1968	64.40	0.72	0.72	0.73	0.74	0.76	0.79	0.84	0.94	1.15	3.93	37.6
Gambia	1992	47.80	0.72	0.74	0.76	0.79	0.83	0.88	0.95	1.07	1.29	2.75	22.2
Gambia	1994	69.20	0.71	0.71	0.71	0.72	0.73	0.76	0.80	0.91	1.23	4.43	49.0
Georgia	1998	50.30	0.71	0.73	0.75	0.78	0.82	0.87	0.96	1.10	1.36	2.78	24.7
Georgia	2002	45.40	0.71	0.73	0.76	0.79	0.84	0.90	0.99	1.13	1.37	2.52	21.5
Germany	1936	49.00	0.71	0.72	0.76	0.76	0.86	0.93	0.98	1.05	1.22	2.85	24.1
Germany	2011	29.00	0.74	0.80	0.84	0.88	0.92	0.97	1.04	1.12	1.25	1.77	8.8
Ghana	1987	35.40	0.73	0.77	0.80	0.84	0.89	0.94	1.01	1.11	1.29	2.06	12.7
Ghana	1998	43.40	0.71	0.72	0.74	0.77	0.82	0.89	0.98	1.12	1.39	2.72	25.0
Greece	1958	38.10	0.73	0.76	0.79	0.83	0.87	0.93	1.00	1.12	1.32	2.17	14.6
Greece	2011	33.50	0.73	0.77	0.81	0.85	0.90	0.97	1.04	1.13	1.28	1.92	11.7
Guatemala	1966	30.00	0.76	0.79	0.82	0.86	0.90	0.95	1.02	1.11	1.27	1.85	9.2
Guatemala	2006	53.00	0.71	0.73	0.74	0.77	0.80	0.85	0.92	1.04	1.29	3.09	27.1
Guinea	1991	48.60	0.71	0.72	0.74	0.77	0.82	0.89	0.98	1.12	1.39	2.72	25.0
Guinea	1994	52.60	0.71	0.72	0.74	0.76	0.79	0.84	0.91	1.04	1.32	3.19	29.3
Guyana	1956	41.90	0.71	0.74	0.78	0.82	0.87	0.93	1.02	1.15	1.38	2.22	17.7
Guyana	1993	53.60	0.71	0.73	0.75	0.77	0.80	0.84	0.89	0.98	1.18	3.31	28.2
Honduras	1968	40.70	0.74	0.76	0.78	0.80	0.84	0.89	0.97	1.08	1.30	2.41	16.5
Honduras	2010	55.00	0.71	0.72	0.73	0.75	0.79	0.85	0.95	1.12	1.43	2.97	29.5
Hong Kong	1971	43.00	0.72	0.75	0.78	0.81	0.84	0.90	0.97	1.09	1.29	2.49	18.2
Hong Kong	2011	53.70	0.71	0.72	0.74	0.77	0.81	0.86	0.95	1.08	1.34	2.98	27.5
Hungary	1955	23.30	0.77	0.82	0.86	0.90	0.94	0.99	1.04	1.12	1.22	1.54	5.7
Hungary	2011	26.80	0.76	0.80	0.84	0.88	0.92	0.97	1.03	1.11	1.24	1.70	7.6
Iceland	2004	24.10	0.76	0.82	0.86	0.91	0.94	0.98	1.03	1.09	1.19	1.64	6.2
Iceland	2011	23.60	0.76	0.84	0.87	0.91	0.94	0.98	1.02	1.09	1.19	1.61	5.9
India	1954	37.60	0.75	0.77	0.80	0.82	0.86	0.91	0.97	1.06	1.24	2.32	14.2
India	2009	27.60	0.77	0.81	0.84	0.87	0.91	0.95	1.00	1.08	1.21	1.83	8.0
Indonesia	1971	46.30	0.73	0.76	0.79	0.81	0.84	0.87	0.91	0.97	1.07	2.96	20.2
Indonesia	1996	39.20	0.73	0.76	0.79	0.82	0.85	0.90	0.97	1.08	1.29	2.35	15.8
Iran	1959	45.50	0.72	0.74	0.76	0.79	0.83	0.88	0.95	1.10	1.36	2.60	20.9
Iran	1973	49.50	0.71	0.73	0.75	0.78	0.79	0.86	0.97	1.06	1.43	2.78	24.8
Ireland	1973	30.00	0.74	0.80	0.84	0.85	0.90	0.95	1.01	1.08	1.31	1.85	9.7

Ireland	2010	33.20	0.73	0.79	0.81	0.84	0.90	0.95	1.02	1.10	1.27	1.98	11.4
Israel	1944	28.50	0.75	0.79	0.83	0.87	0.91	0.97	1.04	1.13	1.28	1.74	8.8
Israel	2007	36.90	0.72	0.74	0.77	0.81	0.86	0.93	1.02	1.15	1.37	2.20	17.0
Italy	1948	42.00	0.72	0.76	0.79	0.82	0.84	0.91	0.96	1.06	1.23	2.52	17.4
Italy	2011	31.90	0.73	0.78	0.82	0.87	0.91	0.97	1.04	1.13	1.27	1.85	10.7
Jamaica	1958	57.70	0.71	0.72	0.73	0.75	0.78	0.83	0.92	1.08	1.42	3.18	31.9
Jamaica	2002	58.00	0.71	0.71	0.73	0.75	0.80	0.85	0.94	1.09	1.39	3.13	32.0
Japan	1956	31.30	0.74	0.77	0.82	0.86	0.90	0.98	1.03	1.13	1.28	1.87	10.7
Japan	2009	31.10	0.74	0.78	0.81	0.85	0.90	0.95	1.01	1.11	1.29	1.95	11.2
Jordan	1973	38.00	0.72	0.76	0.80	0.82	0.82	0.90	0.93	1.04	1.19	2.63	17.9
Jordan	1997	36.40	0.74	0.77	0.80	0.83	0.87	0.92	0.98	1.08	1.25	2.23	13.4
Kazakhstan	1988	25.70	0.76	0.80	0.84	0.88	0.93	0.98	1.05	1.13	1.25	1.62	7.0
Kazakhstan	1996	56.40	0.71	0.72	0.74	0.77	0.80	0.86	0.95	1.11	1.42	2.86	27.2
Kenya	1969	47.90	0.72	0.73	0.75	0.77	0.81	0.88	0.98	1.16	1.50	2.50	22.9
Kenya	2006	44.70	0.72	0.74	0.76	0.79	0.83	0.87	0.96	1.01	1.22	2.87	22.1
Korea, Republic Of	1965	28.50	0.73	0.84	0.85	0.86	0.87	0.98	1.04	1.09	1.31	1.74	8.9
Korea, Republic Of	1998	37.50	0.71	0.75	0.79	0.83	0.94	0.95	1.07	1.18	1.34	1.94	14.4
Kyrgyzstan	1988	26.00	0.80	0.80	0.83	0.86	0.91	0.96	1.03	1.12	1.26	1.66	6.9
Kyrgyzstan	2003	34.20	0.74	0.77	0.79	0.83	0.87	0.92	1.02	1.15	1.33	2.01	12.6
Latvia	1988	22.50	0.77	0.82	0.86	0.90	0.94	0.99	1.05	1.12	1.22	1.50	5.3
Latvia	2012	35.90	0.73	0.77	0.80	0.84	0.88	0.94	1.02	1.13	1.30	2.05	13.1
Lesotho	1986	56.00	0.71	0.72	0.73	0.75	0.79	0.84	0.92	1.06	1.36	3.16	30.0
Lesotho	1995	68.50	0.71	0.71	0.71	0.72	0.74	0.77	0.83	0.99	1.40	3.99	44.9
Lithuania	1988	22.50	0.78	0.83	0.86	0.90	0.94	0.99	1.04	1.11	1.21	1.52	5.3
Lithuania	2011	32.90	0.73	0.78	0.81	0.86	0.91	0.97	1.04	1.14	1.32	1.85	11.3
Luxembourg	1985	25.80	0.76	0.81	0.85	0.88	0.93	0.98	1.04	1.12	1.27	1.61	7.0
Luxembourg	2011	27.20	0.75	0.80	0.84	0.88	0.92	0.97	1.04	1.12	1.24	1.69	7.7
Macedonia, FYR	1994	27.30	0.72	0.74	0.79	0.85	0.94	1.01	1.12	1.23	1.39	1.64	12.6
Macedonia, FYR	2003	32.40	0.71	0.74	0.78	0.85	0.94	1.02	1.11	1.25	1.37	1.69	13.7
Madagascar	1960	56.20	0.72	0.74	0.75	0.77	0.79	0.82	0.87	0.94	1.08	3.51	28.6
Madagascar	2010	39.30	0.73	0.76	0.79	0.82	0.86	0.91	0.97	1.07	1.25	2.37	15.5
Malawi	1969	47.00	0.73	0.75	0.77	0.79	0.82	0.87	0.93	1.03	1.22	2.84	21.2
Malawi	1983	56.70	0.71	0.73	0.74	0.75	0.77	0.81	0.86	0.97	1.26	3.50	31.9
Malaysia	1958	34.80	0.73	0.77	0.80	0.84	0.88	0.94	1.01	1.12	1.30	2.02	12.3
Malaysia	1995	48.50	0.72	0.73	0.75	0.77	0.81	0.86	0.94	1.07	1.35	2.85	24.4
Mali	1989	36.50	0.74	0.76	0.80	0.83	0.87	0.93	1.00	1.10	1.29	2.15	13.5
Mali	1994	78.60	0.71	0.71	0.71	0.72	0.73	0.76	0.82	0.92	1.15	4.49	49.4
Malta	2005	26.90	0.75	0.80	0.84	0.88	0.92	0.97	1.04	1.13	1.27	1.65	7.6

Malta	2011	27.40	0.75	0.80	0.84	0.88	0.92	0.98	1.04	1.11	1.25	1.70	7.9
Mauritania	1987	76.00	0.71	0.71	0.71	0.71	0.72	0.74	0.80	0.93	1.31	4.54	54.1
Mauritania	1993	50.00	0.72	0.74	0.76	0.78	0.81	0.85	0.91	1.00	1.19	3.09	24.3
Mauritius	2007	-	0.78	0.82	0.84	0.88	0.90	0.94	1.00	1.08	1.23	1.80	7.6
Mauritius	2007	38.80	0.74	0.78	0.81	0.84	0.88	0.92	1.00	1.08	1.27	2.12	12.4
Mexico	1957	55.10	0.72	0.73	0.74	0.76	0.77	0.81	0.88	1.00	1.26	3.38	30.0
Mexico	2010	45.00	0.72	0.74	0.77	0.80	0.84	0.89	0.96	1.08	1.31	2.60	20.1
Moldova	1988	24.10	0.77	0.81	0.86	0.90	0.94	0.98	1.04	1.11	1.23	1.57	6.1
Moldova	1997	42.10	0.72	0.75	0.78	0.81	0.86	0.92	1.00	1.12	1.34	2.25	16.4
Morocco	1980	52.40	0.73	0.73	0.73	0.73	0.77	0.80	0.92	1.02	1.49	3.07	28.9
Morocco	1995	35.60	0.72	0.76	0.78	0.82	0.86	0.92	0.99	1.11	1.34	2.24	15.7
Myanmar	1958	38.10	0.74	0.75	0.78	0.81	0.85	0.92	1.02	1.18	1.46	2.01	14.8
Myanmar	2010	-	0.80	0.85	0.88	0.91	0.94	0.98	1.02	1.08	1.18	1.50	4.2
Namibia	1993	74.30	0.71	0.71	0.71	0.72	0.73	0.74	0.78	0.88	1.17	4.65	51.7
Namibia	2010	59.70	0.73	0.74	0.76	0.78	0.79	0.83	0.88	1.03	1.25	3.05	24.4
Nepal	1977	53.00	0.72	0.73	0.75	0.78	0.80	0.84	0.88	0.95	1.14	3.36	27.7
Nepal	2010	32.80	0.72	0.73	0.75	0.77	0.80	0.85	0.93	1.06	1.38	2.88	25.1
Netherlands	1946	50.00	0.71	0.72	0.75	0.79	0.84	0.91	0.97	1.09	1.26	2.80	23.8
Netherlands	2011	25.80	0.75	0.82	0.86	0.89	0.93	0.98	1.03	1.11	1.22	1.67	7.1
New Zealand	1966	31.40	0.75	0.79	0.82	0.86	0.90	0.95	1.02	1.12	1.28	1.88	10.1
New Zealand	1996	40.40	0.72	0.75	0.79	0.82	0.86	0.92	1.00	1.14	1.36	2.21	16.3
Nicaragua	1993	50.30	0.72	0.73	0.75	0.78	0.81	0.86	0.94	1.06	1.31	2.90	24.5
Nicaragua	2005	50.00	0.72	0.73	0.75	0.78	0.81	0.87	0.94	1.05	1.28	2.92	24.3
Niger	1992	36.10	0.74	0.77	0.80	0.84	0.87	0.92	0.99	1.08	1.24	2.20	13.2
Niger	1995	50.60	0.71	0.72	0.74	0.77	0.82	0.90	1.01	1.17	1.44	2.59	25.2
Nigeria	1980	42.60	0.71	0.73	0.74	0.76	0.80	0.86	0.95	1.13	1.43	2.77	25.5
Nigeria	1997	50.60	0.72	0.73	0.75	0.78	0.81	0.86	0.93	1.04	1.26	2.97	24.6
Norway	1957	40.00	0.71	0.75	0.78	0.84	0.90	0.99	1.05	1.16	1.30	2.08	15.7
Norway	2011	22.90	0.76	0.84	0.88	0.91	0.95	0.99	1.04	1.09	1.19	1.56	5.7
Pakistan	1970	14.60	0.86	0.89	0.91	0.93	0.95	0.98	1.01	1.05	1.12	1.39	2.4
Pakistan	1996	30.60	0.75	0.78	0.80	0.82	0.84	0.87	0.91	0.98	1.10	2.73	17.0
Panama	1960	50.00	0.72	0.74	0.76	0.78	0.81	0.86	0.92	1.03	1.23	2.99	23.9
Panama	2010	49.00	0.71	0.73	0.75	0.78	0.82	0.88	0.95	1.07	1.33	2.80	23.8
Paraguay	1983	45.10	0.72	0.74	0.76	0.80	0.83	0.88	0.97	1.11	1.38	2.52	20.1
Paraguay	2010	50.00	0.71	0.73	0.75	0.78	0.82	0.87	0.95	1.06	1.28	2.87	24.2
Peru	1961	57.00	0.71	0.72	0.72	0.74	0.77	0.82	0.89	0.99	1.29	3.55	34.6
Peru	2010	45.00	0.72	0.74	0.76	0.80	0.84	0.90	0.99	1.11	1.35	2.49	20.0
Philippines	1957	49.20	0.72	0.73	0.75	0.78	0.81	0.86	0.94	1.07	1.33	2.84	23.5
Philippines	2009	44.80	0.72	0.74	0.76	0.78	0.82	0.88	0.96	1.10	1.37	2.59	20.9

Poland	1956	27.00	0.76	0.81	0.84	0.88	0.92	0.97	1.03	1.11	1.25	1.71	7.6
Poland	2011	31.10	0.74	0.79	0.82	0.86	0.91	0.96	1.02	1.11	1.26	1.86	10.0
Portugal	1980	32.00	0.74	0.78	0.82	0.84	0.93	0.96	1.02	1.12	1.29	1.85	10.2
Portugal	2011	34.20	0.74	0.78	0.81	0.85	0.89	0.94	1.00	1.10	1.27	2.05	12.0
Puerto Rico	1953	41.50	0.72	0.75	0.78	0.80	0.86	0.91	0.95	1.05	1.39	2.43	18.1
Puerto Rico	1977	39.70	0.72	0.75	0.78	0.82	0.86	0.93	1.01	1.15	1.39	2.14	15.9
Romania	1989	23.70	0.76	0.82	0.86	0.91	0.95	0.99	1.04	1.12	1.23	1.52	5.8
Romania	2011	33.20	0.72	0.77	0.81	0.86	0.91	0.98	1.05	1.15	1.32	1.83	11.5
Russian Federation	1988	23.80	0.77	0.82	0.86	0.90	0.94	0.99	1.05	1.12	1.23	1.55	6.0
Russian Federation	2000	45.60	0.71	0.73	0.77	0.81	0.86	0.91	1.00	1.08	1.31	2.53	20.5
Senegal	1960	58.70	0.71	0.72	0.73	0.75	0.78	0.82	0.88	1.01	1.26	3.45	32.2
Senegal	1994	41.30	0.73	0.76	0.78	0.81	0.85	0.90	0.96	1.07	1.26	2.47	16.9
Serbia and Montenegro	1968	17.90	0.81	0.86	0.89	0.92	0.95	0.99	1.04	1.09	1.18	1.39	3.4
Serbia and Montenegro	2001	37.80	0.76	0.80	0.84	0.88	0.92	0.96	1.02	1.09	1.22	1.81	8.3
Sierra Leone	1968	44.00	0.72	0.75	0.78	0.79	0.82	0.86	0.94	1.04	1.26	2.76	20.8
Sierra Leone	1989	62.90	0.71	0.71	0.71	0.71	0.75	0.84	0.99	1.21	1.57	3.16	39.1
Singapore	2008	47.40	0.72	0.74	0.78	0.82	0.87	0.95	1.03	1.15	1.37	2.13	16.3
Singapore	2012	47.80	0.72	0.74	0.78	0.82	0.88	0.94	1.03	1.15	1.32	2.19	16.4
Slovak Republic	1988	19.50	0.80	0.85	0.88	0.91	0.95	0.99	1.04	1.09	1.18	1.46	4.1
Slovak Republic	2011	25.70	0.75	0.81	0.86	0.90	0.93	0.98	1.04	1.11	1.22	1.64	7.0
Slovenia	1987	21.50	0.78	0.82	0.86	0.89	0.93	0.98	1.03	1.11	1.21	1.58	5.8
Slovenia	2011	23.80	0.76	0.82	0.87	0.91	0.95	0.99	1.04	1.11	1.21	1.57	6.1
South Africa	1965	58.10	0.71	0.71	0.72	0.74	0.77	0.82	0.93	1.14	1.65	2.98	33.5
South Africa	2008	59.40	0.71	0.71	0.72	0.72	0.73	0.75	0.80	0.93	1.39	4.14	46.2
Spain	1965	38.90	0.73	0.76	0.79	0.82	0.86	0.91	1.00	1.12	1.32	2.22	15.2
Spain	2011	34.00	0.72	0.77	0.81	0.86	0.91	0.97	1.05	1.16	1.33	1.83	12.2
Sri Lanka	1953	48.30	0.72	0.74	0.76	0.80	0.84	0.86	0.92	1.01	1.17	2.96	22.4
Sri Lanka	2002	47.00	0.72	0.74	0.76	0.79	0.82	0.88	0.94	1.06	1.30	2.74	21.7
Sudan	1963	44.60	0.72	0.74	0.76	0.79	0.83	0.89	0.98	1.13	1.40	2.44	19.7
Sudan	1968	44.00	0.71	0.73	0.76	0.79	0.84	0.93	1.00	1.13	1.27	2.55	20.7
Sweden	1954	38.00	0.72	0.75	0.79	0.83	0.90	0.97	1.03	1.14	1.30	2.06	14.2
Sweden	2011	24.40	0.75	0.82	0.86	0.91	0.95	0.99	1.05	1.11	1.22	1.57	6.3
Switzerland	1982	35.10	0.72	0.78	0.83	0.86	0.90	0.95	1.00	1.09	1.22	2.11	13.0
Switzerland	2011	29.70	0.74	0.80	0.83	0.87	0.91	0.97	1.03	1.10	1.23	1.84	9.2
Syria	2004	35.80	0.74	0.77	0.80	0.83	0.87	0.92	0.99	1.09	1.28	2.16	13.0
Syria	2007	32.00	0.76	0.79	0.82	0.86	0.89	0.93	1.00	1.09	1.25	1.95	9.9
Taiwan	1953	57.60	0.71	0.72	0.73	0.75	0.78	0.82	0.91	1.05	1.37	3.27	31.7
Taiwan	2005	30.50	0.75	0.78	0.82	0.85	0.89	0.94	1.01	1.11	1.28	1.94	10.6
Tanzania	1967	50.30	0.72	0.74	0.76	0.78	0.81	0.85	0.91	1.02	1.22	3.03	24.1

Tanzania	1993	39.50	0.72	0.74	0.76	0.78	0.82	0.87	0.95	1.07	1.33	2.74	22.4
Thailand	1962	41.30	0.76	0.77	0.77	0.77	0.78	0.88	0.92	1.17	1.30	2.53	18.6
Thailand	2011	-	0.73	0.76	0.79	0.81	0.84	0.90	0.96	1.04	1.24	2.53	17.3
Trinidad And Tobago	1971	45.00	0.72	0.73	0.76	0.80	0.83	0.95	0.99	1.11	1.39	2.43	20.2
Trinidad And Tobago	1992	49.50	0.71	0.73	0.75	0.79	0.82	0.88	0.96	1.10	1.41	2.68	23.9
Tunisia	1961	46.00	0.72	0.73	0.76	0.79	0.83	0.89	0.99	1.13	1.41	2.49	20.9
Tunisia	1990	40.20	0.73	0.75	0.78	0.82	0.86	0.92	1.00	1.11	1.31	2.28	16.1
Turkey	1968	56.80	0.71	0.72	0.73	0.75	0.78	0.83	0.91	1.05	1.33	3.24	30.5
Turkey	2006	44.80	0.72	0.74	0.77	0.80	0.85	0.90	0.98	1.10	1.34	2.50	19.8
Turkmenistan	1988	26.40	0.79	0.80	0.83	0.86	0.91	0.96	1.03	1.12	1.26	1.68	7.1
Turkmenistan	1993	35.80	0.73	0.76	0.80	0.84	0.88	0.94	1.02	1.13	1.33	2.03	13.0
Uganda	1970	26.60	0.77	0.82	0.86	0.89	0.92	0.96	1.00	1.06	1.17	1.83	7.5
Uganda	2010	42.30	0.73	0.75	0.78	0.81	0.85	0.89	0.95	1.05	1.27	2.55	17.9
Ukraine	1988	23.30	0.77	0.82	0.86	0.90	0.94	0.99	1.04	1.11	1.22	1.55	5.7
Ukraine	1996	32.50	0.73	0.77	0.81	0.85	0.90	0.94	1.01	1.11	1.27	2.04	12.1
United Kingdom	1960	35.50	0.73	0.76	0.80	0.84	0.89	0.95	1.03	1.15	1.35	1.96	12.9
United Kingdom	2011	33.00	0.74	0.78	0.82	0.85	0.89	0.95	1.01	1.11	1.26	1.97	11.2
United States	1972	38.10	0.72	0.74	0.78	0.83	0.90	0.97	1.07	1.18	1.35	2.01	15.5
United States	2010	37.30	0.71	0.75	0.78	0.82	0.87	0.93	1.02	1.14	1.36	2.21	16.9
Uruguay	1961	36.60	0.73	0.76	0.79	0.82	0.86	0.91	0.99	1.09	1.29	2.28	15.2
Uruguay	2010	43.00	0.72	0.75	0.77	0.80	0.84	0.89	0.97	1.10	1.35	2.45	18.4
USSR	1980	24.50	0.76	0.81	0.85	0.90	0.94	0.99	1.05	1.13	1.25	1.55	6.4
USSR	1989	28.90	0.75	0.80	0.84	0.88	0.92	0.98	1.03	1.12	1.23	1.73	8.0
Uzbekistan	1989	28.20	0.76	0.80	0.83	0.87	0.92	0.97	1.03	1.12	1.26	1.73	8.2
Uzbekistan	2001	47.20	0.71	0.73	0.75	0.79	0.83	0.89	0.98	1.12	1.38	2.60	22.7
Venezuela	1962	43.80	0.72	0.74	0.77	0.80	0.84	0.90	0.99	1.11	1.35	2.44	19.0
Venezuela	2010	36.00	0.73	0.76	0.80	0.84	0.88	0.95	1.03	1.14	1.32	2.03	13.4
Vietnam	1993	33.40	0.75	0.78	0.81	0.84	0.87	0.92	0.99	1.10	1.29	2.08	12.1
Vietnam	1998	35.40	0.74	0.77	0.80	0.82	0.86	0.91	0.97	1.08	1.30	2.24	14.2
Yemen	1992	39.50	0.73	0.75	0.79	0.82	0.87	0.92	0.99	1.09	1.29	2.29	15.6
Yemen	1998	21.80	0.76	0.81	0.86	0.91	0.97	1.02	1.08	1.16	1.24	1.36	5.3
Zambia	1959	52.30	0.73	0.73	0.75	0.76	0.79	0.83	0.89	1.00	1.23	3.19	26.2
Zambia	2004	50.00	0.71	0.71	0.72	0.75	0.77	0.88	0.97	1.33	1.67	2.62	32.1
Zimbabwe	1968	66.30	0.72	0.72	0.72	0.73	0.75	0.78	0.83	0.92	1.12	4.09	39.6
Zimbabwe	1995	70.30	0.71	0.71	0.72	0.72	0.73	0.75	0.78	0.84	1.00	4.82	51.3

RII: Refractive Inequality Index (subscripts denote income groups or strata from the lower end), RLI: Refractive Lorenz Index
Source: Gini coefficient - WIID3.0b), September 2014; Self-elaboration, otherwise