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A Note on Methodology of Treating Income in Human Development Index[1]

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The methodology of discounting income has evolved significantly over time to capture true contribution of income to human development. We critically review all the discounting schemes adopted so far with respect to the use of poverty line or threshold income and realise that the spirit of Atkinson's formula has been the most sensible and purposeful in operationalising the idea that income above certain limit has diminishing contribution to human well-being. However, as the formula was replaced by a simple logarithmic scheme reaping some mathematical advantages, we propose an alternative one, which maintains the spirit of the former and the essential technicalities of the latter. Further, we find that although the present United Nations Development Programme (UNDP) method does not use any poverty line in a direct manner, mathematically it works with one, which is nothing but the base of the logarithm. The new proposed method works with robustness and more clarity than the others.

I. Introduction

The methodology of discounting income in Human Development Index (HDI) has evolved significantly over time to capture true contribution of income to human development. It is commonly understood that one does not need unlimited income for achieving a decent standard of living. This idea is reflected in the methodology of discounting income, which is based on the assumption that income above a specified poverty line or threshold level has diminishing contribution to human development. Considering this phenomenon of diminishing marginal utility of income, the creator of HDI, the UNDP, in its first Human Development Report (HDR) in 1990, used common logarithm to transform income levels into well-being scores within a specified range, where the upper limit has been the threshold income (UNDP, 1990: 13). Although the method

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assumed diminishing returns in the conversion of income into the fulfilment of human needs, it did not recognise the contribution of income to human development beyond the threshold income. The method was refined meaningfully when UNDP (1991: 89-91) decided to give a progressively lower weight to income beyond the poverty line, rather than the zero weight previously given. Consequently, a poverty line or threshold income was set and instead of taking logarithm of income levels, Atkinson's formula (UNDP, 1991: 90) was adopted for discounting income in between the threshold level and a specified upper limit. It was assumed that income above the (specified) upper limit has no contribution to human development. The subsequent HDRs till 1998 (UNDP, 1991-1998) used the same formula with varied range of income and threshold level as summarised in Table 1.

The method of treating income is revolved again in 1999 in response to the observations made by Anand and Sen (2000)[2] on the income component of HDI. They observed that in the case of Atkinson's formula (1970), the function does not satisfy essential mathematical properties.[3] Consequently, as per the recommendations made by Anand and Sen (1997: 1-4), UNDP (1999: 159) returned to the initial method of logarithmic transformation with some modification, but dropped the concept of poverty line or threshold income. Under this new methodology, all observed income levels are transformed into figures to be comprehended as well-being scores within a defined range of income. This new conversion scheme is being followed continuously for more than a decade with some changes in 2010 and 2011, where instead of common logarithm, natural logarithms were used with significant extension of range of income (UNDP, 2010: 216; UNDP, 2011: 168) as shown in Table 1.

By adopting this discounting scheme, UNDP (1999: 159) confirms concavity of the function as well as smooth conversion throughout the income range, which does not discount income as severely as the formula used earlier and where the asymptote starts quite late[4] without penalising the middle-income countries disproportionately.

The main objective of this methodological note is to examine whether conversion of income at various levels, particularly for higher and lower income countries, conforms to the (above-mentioned) claims made by UNDP. However, examining the extent of discount (or whatever it may be after conversion) is simply not possible with the simple logarithmic scheme, as the units of measurement before and after conversion are not same. More

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2. Appeared first as Anand and Sen (1999).

3. The function is not concave and the elasticity is neither monotonic increasing nor monotonic decreasing for income y y ≥ 0, as discussed later.

4   May be confirmed from figure TN1 in HDR 1999 (UNDP, 1999: 159). Incidentally, as table TN1 of the same report (pp. 164-7) does not display index of gross domestic product (GDP), additionally we compute the same using the old and new formulae for 1998 and reproduce figure TN1. We see that the asymptote in the former starts when income level reaches around US$ 6000 (purchasing power parity; PPP) and such a level corresponds to middle-income countries like Brazil and Costa Rica. On the contrary, it starts quite late when income level tends to reach the maximum level in the case of the latter.

clearly, the formula does not readily give discounted income levels—rather it gives figures to be comprehended as well-being scores. In order to make this possible, our plan is to derive an alternative formulation equivalent to the UNDP one, which will work exactly in a similar fashion and additionally express converted figures in unit of income.

Table 1

Poverty Line and Income Range in HDI: 1990-2011

Year	Method of Treating Income	Poverty Line* (US$ PPP)	Income Range (US$ PPP)	
			Min	Max
1990	Common logarithm	4861	220	4861
1991		4829	350	19850
1992		4829	380	20998
1993		4829	367	21449
1994	Atkinson's formula	5120	200	40000
1995		5120	100	40000
1996		5711	100	40000
1997		5835	100	40000
1998		5990	100	40000
1999-2009	Common logarithm	absent	100	40000
2010	Natural logarithm	absent	163	108211
2011		absent	100	107721

Note: * Or threshold income (average income of industrial countries till 1994; global average thereafter till 1998).

Source: HDRs, 1990-2011.

However, it is to be noted that although our goal is to reflect through the conversion, how income of an individual makes one capable of improving the levels of achievements in the dimensions of quality of life other than life expectancy and literacy, or (in the context of a cross-country comparison) how per capita income of the country reflects the capability of the country in improving the level of achievements of its citizens in the dimensions of quality of life other than life expectancy and literacy, knowing the extent of discount is a methodological requirement. Neither the figures after simple logarithmic transformation nor the discounted income levels convey any significant meaning (in terms of well-being) unless they are further converted and ranked with the modalities of an ordinal scale. In the proposed case, if we get discounted figures in units of income, we will be able to focus on quality of life finally with the additional feature of examining about how the method penalises the countries in which income exceeds the threshold level. The latter is

important, as UNDP (1999: 159) reports that the old method reduces the US$ 34,000 (PPP$) between the threshold and maximum level of income to a mere $321 (PPP$). And, according to them, in many cases, income lost its relevance as a proxy for all dimensions of human development other than a long and healthy life and knowledge. Further, as we begin with an assertion that one does not need unlimited income for achieving a decent standard of living, we need a method from which we will be able to relate about how much income (relative to a poverty line) is actually necessary to maintain a certain standard of living.[5]

The issue of knowing income levels after conversion is not only an implicit requirement of this paper. If someone is interested to know actual discounted income figures, it probably highlights one possible limitation of the present discounting scheme of UNDP. Since readers are unlikely to compare 'logs' of incomes,[6] it is better to simply use the absolute[7] levels (Rao, 1991: 1455). Atkinson's formula was also questioned on this issue. Anand and Sen (2000) pointed out that, in true sense, discounted figures in Atkinson's formula are not to be comprehended as dollar figures. In such a situation, our new proposed method is likely to provide with a solution.

Income is discounted with respect to a poverty line or threshold income. When discounted, the figures or levels are not absolute,[8] rather relative in true sense of the term. However, for expressing the discounted figures in unit of income, there is an innate requirement of the concept of poverty line. Conversely, there is no consensus in literature on the question of using and/or omitting poverty line in income discounting schemes. Theoretically, while justifying the need for the income component of HDI as an indirect indicator for many other capabilities (beyond longevity and education), Anand and Sen (2000: 86) feel that there are many important capabilities which are critically dependent on one's economic circumstances. Thus, the income level enjoyed by people, "especially close to poverty lines, can be very crucial information on the causal antecedents of basic human capabilities." This tacitly confirms the need for the income component intuitively with the concept of poverty line in studies of human well-being and development. Empirically, the need for the concept of poverty line or threshold income has been emphasised by contemporary authors like Chaubey (1998; 2002) and Bhatnagar (2001). On the contrary, it is now often argued that in spirit of human development (and capability approach), neither discounted income figures nor poverty line are important as the methodology intuitively no longer focuses on income, but on what income does to human well-being. Such argument seems to support the present UNDP methodology of

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5 We will comment later in the context of results in Tables 5 and 6.

6 Although log of income has the same ranking as incomes, being a monotonous function of income also shows a diminishing marginal contribution of income to well-being, unlike income itself.

7 One may comprehend it as 'relative' in true sense of the term, which is discussed later.

8 Please refer the previous footnote.

discounting income, as visibly it works in the absence of poverty line. However, by introducing the proposed alternative formulation, we would like to show that although UNDP presently does not use any poverty line in a direct manner, mathematically it works with one, which is nothing but the base of the logarithm.

Finally, one may even work outside the HDI framework and may be interested to discount income with respect to a poverty line or threshold income. There should be a method to serve the purpose. In response to all these requirements, we feel that the spirit of Atkinson's formulation has been the most sensible and purposeful in operationalising the idea that income above certain limit has diminishing contribution to human well-being. This further enhances our desire for an alternative formulation, which will work with the spirit of the Atkinson's formula and satisfy the necessary technicalities of the simple logarithmic scheme.

II. A Closer Look to Various Income Discounting Schemes Adopted in HDRs

The first HDR (UNDP, 1990: 109) adopted common logarithm (the base of which is 10) to discount income in between a lower limit and threshold income (as shown in Table 1). This can be presented as:

$$W(y) \; = \; \log y \qquad\qquad \text{for } 0 < y \le y^* \qquad\qquad\qquad (1a)$$
$$\quad = \; \log y^* \qquad\qquad \text{for } y \ge y^*, \qquad\qquad\qquad (1b)$$

where y is income, y^* is poverty line or threshold income, and $W(y)$ is comprehended as well-being corresponding to income y.

In order to recognise contribution of income (to human development) above the threshold income, UNDP (1991: 89) adopted Atkinson's formulation for the utility of income [see also Anand and Sen (2000: 88-9)]:

$$W(y) = \frac{1}{1 - \varepsilon} \times y^{1 - \varepsilon},$$

where ε is the elasticity of the marginal utility of income with respect to income, which measures the magnitude of diminishing returns in conversion. The value of ε lies between 0 and 1. When ε = 0, there is no conversion:

$$W(y) \; = \; y.$$

UNDP has set ε = 0 for all countries with income less than or equal to threshold level, $y \le y^*$. The value of ε is assumed to increase gradually with income above the threshold level, $y > y^*$. Finally, when ε →1,

$$W(y) \; = \; \log y.$$

In other words, Atkinson's formula has been configured to work with variable elasticity of the marginal utility of income. Consequently, different values of ε are set for different income groups as shown in Table 2.

Table 2

Values of Elasticity in Atkinson's Formula (1991-1998)

Value	Income groups				
of	$0 \leq y \leq y^*$	$y^* \leq y \leq 2y^*$	$2y^* \leq y \leq 3y^*$	...	$ay^* \leq y \leq (a+1)y^*$
ε	0	1/2	2/3	...	$a/(a+1)$

Note: a: Constant, 1, 2, 3, ...

The above configuration gives us a multi-step utility function (UNDP, 1991: 90):

$$W(y) = y \qquad \text{for } 0 \leq y \leq y^* \qquad (2a)$$

$$= y^* + 2(y - y^*)^{1/2} \qquad \text{for } y^* < y < 2y^* \qquad (2b)$$

$$= y^* + 2(y^*)^{1/2} + 3(y - 2y^*)^{1/3} \qquad \text{for } 2y^* < y < 3y^* \qquad (2c)$$

and so on.

Although the formula has been used during 1991-1998, Anand and Sen (2000) found that in case of it:

(1) the function is piecewise concave: it is not concave throughout its range and hence, it does not reflect diminishing returns in the contribution of income to human development, and

(2) elasticity of the marginal utility of income with respect to income does not work essentially for a progressive diminishing pattern meaning the higher the income relative to the poverty level, the less sharply (instead of more sharply) the diminishing returns affect the contribution of income to human development.

In order to avoid such issues with the multi-step utility function and/or variable elasticity valuation function, UNDP (1999: 159) adopted common logarithm[9] for treating all income levels within a specified range (as shown in Table 1). The formula used during 1999-2009 was:

$$W(y) = \log y \qquad \text{for } 100 \leq y \leq 40000. \qquad (3a)$$

As discussed above, this simple logarithmic scheme is a special case of Atkinson's formulation for the utility of income, where the elasticity of the marginal utility of income with respect to income (ε) is constant with a value close to one (ε → 1).

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9 In response to recommendations made by Anand and Sen (1997).

From 2010 onwards, UNDP (2010, p. 216; 2011: 168) replaced the common logarithm in the discounting scheme by natural logarithm (the base of which is e or 2.718). The formulae used in 2010 and 2011 are, respectively:

$W(y) = \ln y$ for $163 \leq y \leq 108211$,   (3b)

and

$W(y) = \ln y$ for $100 \leq y \leq 107721$,   (3c)

where ln stands for natural logarithm.

III. Alternative Methods of Discounting Income

In order to rectify the issues associated with (i) piecewise concavity of the function derived from Atkinson's formulation and (ii) undesired functionality of the variable elasticity valuation function, Anand and Sen (2000: 93-4) proposed two classes of valuation function up to a positive affine transformation. First,

$$W(y) = -e^{-\gamma y}, \tag{4a}$$

where a m 0 and gy is the elasticity component, which starts at 0 when y = 0 and increases linearly with income. Second,

$$W(y) = -y^{-\alpha}.e^{-\gamma y}, \tag{4b}$$

where a m 0 and g m 0. This is a more general class of valuation function, which combines the constant relative and constant absolute inequality aversion forms. Both of the above functions are increasing, concave throughout the income range, and have a variable and increasing elasticity of marginal valuation (Anand and Sen, 2000). It is to be noted that although the latter of these two expresses the discounted figures in units of income, it is applicable for the whole range of income without the use of the concept of poverty line.[10]

Chaubey (1998; 2002) suggested another alternative discounting scheme within the spirit of Atkinson's formulation, which recognises the use of poverty line or threshold income and the phenomenon of diminishing contribution of income above that. In his formulation:

$W(y) = y$ for $0 \leq y \leq y^*$ (5a)

$\quad\quad = y^*+y^* \log(y/y^*)$ for $y \geq y^*$.   (5b)

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10 While recommending the different formulations, Sen was inclined to go for the simplicity of a 'constant elasticity' one. Although, the case of 'variable elasticity' was of great theoretical interest, particularly for keeping the 'absolute inequality aversion' constant, he did not recommend it to the UNDP, as it appeared little harder to grasp and might be somewhat more difficult to explain to the lay readers of the HDR. This may be a practical reason for which UNDP did not use this formula. [Please see the letter written by Sen to HDR Office, which is attached at the end of the above-mentioned Anand and Sen (1997)].

Bhatnagar (2001) proposed another formulation, which is an amalgamation of the method proposed by Chaubey (1998; 2002) and Atkinson's formula adopted by UNDP (1991: 90) with some modification. He classified different income groups as: $0 \leq y \leq y^*$, $y^* \leq y \leq 2y^*$, $2y^* \leq y \leq 4y^*$, and so on [in general: $2ny^* \leq y \leq (n+1)y^*$] and proposed the following:

$$W(y) \ = y \hspace{3cm} \text{for } 0 \leq y \leq y^* \hspace{1cm} (6a)$$

$$= y^* + y^* \log(y/y^*) \hspace{1.5cm} \text{for } y^* \leq y \leq 2y^* \hspace{1cm} (6a)$$

$$= y^* + (1/2)y^* \log(2) + (1/2)y^* \log(y/y^*) \ \text{for } 2y^* \leq y \leq 4y^* \hspace{1cm} (6c)$$

and so on.

In general:

$$W(y) = \left(\sum\nolimits_{m=0}^{n} (1/2^m)y^* \log(2^m) + (1/2^n)y^* \log(y/y^*) \right) \text{for } 2ny^* \leq y \leq (n+1)y^* \text{ and } n \geq 1.$$

It is claimed to be a better formulation than the Atkinson's one in terms of a desired diminishing pattern without sacrificing the concept of poverty line where the discounted figures are free from 'units and dimensions'[11] (Bhatnagar, 2001: 56-7). Suggestions of Chaubey (1998; 2002) and Batnagar (2001) also yield transformations which are in the units of income and have inequality aversion equal to unity. However, those are subject to other limitations (piecewise concavity, etc.) of the multistep utility functions as applicable for Equations (2a) to (2c). As a result, although such formulae provided with meaningful alternatives, these are yet to be popularised (Nayak, 2009: 9).

IV. Need for the Proposed Alternative

Although our search for alternative income discounting schemes may not be exhaustive, all the schemes presented above include both constant and variable elasticity valuation functions. However, a meaningful alternative to the present UNDP formulation, within the framework of constant elasticity valuation function, is not known. While proposing the alternative formulation, Anand and Sen (2000: 93) recognises the intention of UNDP towards configuring the elasticity function ["the higher the income relative to the poverty level, the more sharply the diminishing returns affect the contribution of income to human development" (UNDP, 1991: 90, etc.)]. However, practically, their formulation is applicable for the whole range of income in order to ensure the possible full workability of the

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11, The principle of 'units and dimension' is borrowed by the author from the theory of physics. For example, in his view in Equation (2c) of this paper, the unit of measurement for the first term is dollars. The unit of measurement for the second term is square root of dollars. Similarly, the unit of measurement for the third term is cube root of dollars and so on. Algebraic operations involving addition of terms carrying non-homogenous units are not tenable (Bhatnagar, 2001: 50-1).

elasticity function in its complete range:[12] 0-1. In other words, one may need to sacrifice the use of poverty line while working with it in order to ensure concavity throughout. By definition, the remaining two (Equations 5d and 6b-6d) are applicable for income above the poverty line. This clearly makes a scope for us to derive an alternative formulation within the framework of constant elasticity valuation function, which will be applicable for the whole range of income or part of it as well (say, for $y > y^*$) with simplicity in application.

V. The New Proposed Method of Treating Income

Following the analysis of previous sections and using previously defined notations, we may assert that 'ln y' gives a very smooth conversion with concavity throughout for all $y > 0$. If we want the same pattern of conversion relative to a poverty line or threshold income, we may divide 'ln y' by 'ln y^*'. Now, if the expression is multiplied by threshold income y^*, it gives converted income levels (in units of income[13]) relative to the poverty line. This can be expressed as:

$$I = y^* (\ln y / \ln y^*), \qquad\qquad (7a)$$

where I is converted income. If applied for the whole income range, this formula works exactly in a similar fashion as the present UNDP method does. Further, it proves to be a good substitute of Atkinson's formulation if applied for $y > y^*$. The only point of diversion from the methodology in Atkinson's formulation is that while the former works with a variable elasticity valuation function, the latter works with a constant elasticity one.

It is a fact that the above formulation gives a welfare function which has a relative inequality aversion equal to unity. If we follow Anand and Sen (2000), it is an undesirable property. However, if we follow footnote 10, as above, it does pose any series threat to the application of the method as long as it is applied to the whole range of income. This may also be addressed multiplying the above by a component with increasing inequality aversion:

$$I = -y^* (\ln y / \ln y^*) \cdot e^{-\gamma y}, \qquad\qquad (7a^*)$$

where $\gamma \geq 0$.

VI. Equivalence of the New Method with the Present UNDP One

One of the basic properties of logs says:

$$\log_b x = \log_a x / \log_a b. \qquad\qquad (7á)$$

12, Anand and Sen (2000: 94) have shown that in their formulation, elasticity starts at 0 when $y = 0$ and increases linearly with income y.

13 Things for which we cannot comprehend 'ln y' or 'ln y^*' as income are present in both the numerator and denominator and hence they are wiped out.

If we follow this rule and ignore the constant y^* outside the parentheses in the right-hand side of the previous expression (Equation 7a), we have:

$$\hat{I} = \log_{y^*}(y) \qquad\qquad (7b)$$

where $\hat{I}$ is well-being score, which is nothing but logarithm of income with base being the poverty line or threshold income. Mathematically, the pattern of conversion of the above scheme is equivalent to that of the present UNDP method (Equation 3c):

$$W(y) = \log_e(y). \qquad\qquad (7c)$$

It is needless to say that both the above formulae convert income exactly in a similar fashion even if $y^* \neq e$. Well-being scores may differ from one logarithmic scheme to another for different base, but the patterns are perfectly correlated. It may also be noted (it follows mathematically) that if the logarithmic schemes (Equations 7b and 7c) are multiplied by their respective bases, then also they are perfectly correlated. As above, the proposed new method (Equation 7a) may be rewritten as:

$$I = y^* \log_{y^*}(y). \qquad\qquad (7d)$$

It says that the proposed new method of conversion is: threshold income multiplied by logarithm of income with base being the threshold income. This result makes us to realise that, in the new UNDP scheme (Equations 3a-3c), 10 or e or in general the base of the logarithm works as poverty line, relative to which conversion of income is done at all levels.

As it is equally applicable for income above the poverty line, we may present the utility or welfare function as:

$$I = y \qquad\qquad \text{for } 0 \leq y \leq y^* \qquad\qquad (7e)$$

$$= y^* \log_{y^*}(y) \qquad\qquad \text{for } y > y^*. \qquad\qquad (7f)$$

Following standard procedures of computing dimension indices (UNDP, 2011: 167-8) and utilising data from HDR 2011 on gross national income (GNI), it is confirmed that converted figures from schemes 7b and 3c (or 7c), and converted incomes from scheme 7d, assuming $y^* = $ US\$ 10082 PPP, the world average income as appeared in HDR 2011 (UNDP, 2011: 130) in 7b and 7d, produce exactly the same index of income for all the 187 countries (as it should be in HDR 2011).[14]

VII. Fulfilling the Objectives

By introducing the new method, we have nearly fulfilled all the objectives, except the main one—examining the extent of discounts at various levels. Before that we have to comment on two issues: first, duality in conceptualising the notion of poverty line and second, whether the term 'discount' is appropriate, as we would like to show that, instead

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14. Index of income is not separately displayed in HDR 2011.

of marking down, the so-called new income discounting scheme of UNDP puts up income levels at the lower end. We discuss the first issue and keep the second one for empirical examination.

As the proposed new method (Equation 7d) expresses converted figures in units of income, can we comprehend the converted figures from the following formula as income:

$$W = e \ln y, \tag{8}$$

where e =2.718? Obviously we cannot, as US$ 2.718 PPP cannot be considered as poverty line or threshold income in reality. It was customary for UNDP to consider the global average income as the poverty line or threshold level during 1995-1998. Depending on the level and scope of the study, a threshold income may be defined locally or globally relying on relevant public reasoning and democratic procedures.[15] For example, if the index of GDP is to be computed for the states in India, we need to fix a well-acceptable threshold income as per local conditions. Similarly, if we are to do the exercise for a region, say Asia, we need to think accordingly. However, in no way it will come down to a figure equal to e or 2.718 in reality. At this point, we classify the poverty lines (used by UNDP so far) in two categories. First, the *de jure* one: a consideration on threshold income (obeying the above-mentioned principle) beyond which it is supposed to have diminishing contribution to human development. Second, the *de facto* one: a mathematical poverty line, such as the base of logarithm or the ones used while applying Atkinson's formula.

Table 4

Different Concepts of Poverty Line

Year	Poverty line	
	De jure	*De facto*
1990	Averages of industrial countries*	10
1991-1994	Averages of industrial countries*	Averages of industrial countries*
1995-1998	Global averages*	Global averages*
1999-2009	Absent	10
2010-2011	Absent	e or 2.718

Note: * As shown in Table 1.

Source: HDRs, 1990-2011.

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15 As per Sen's priorities (2004) on similar issues. For example, Sen has not specified an exact and definite list of capabilities and functionings. Such a selection, according to him, will always have to be the object of discussion, either through some deliberative democratic procedure or by providing social scientific and political space to allow for different selections which can then be discussed. In the latter case, the social scientists could perform a sensitivity analysis with regard to the selection of the particular criterion having the relevance (see also Robeyns, 2000).

Table 4 summarises information on the use of different poverty lines by UNDP so far. For example, the first HDR in 1990 worked with a dual concept of poverty line with US$ 4,861 PPP as *de jure* and 10 as *de facto* one. During the period of Atkinson's formulation, the *de jure* consideration perfectly matched with the *de facto* one. This should also be a desired property in order to make the proposed new method (or any equivalent one) workable and reasonably acceptable.

Table 5 shows income conversion under the new proposed scheme (Equation 7d) and Atkinson's formula (2a-2c, etc.) as well as proportion of converted income (or whatever it may be in Atkinson's formulation) to its respective GNI for some selected countries. In case of the former, we adopted four different schemes with four different poverty lines. However, it is to be noted that the patterns of discounts are perfectly correlated to each other and they produce same index of income for the countries. We begin with the two extreme cases. First, income is discounted with respect to that of the poorest country (US$ 265 PPP in Liberia as in HDR 2011). We see that although such a scheme discounts income smoothly at the lower end, it seriously disregards the interest of the high income countries. Second, at the other extreme, we convert income with respect to that of the richest country (US$ 1,07,721 PPP in Qatar as in HDR 2011). Although, it looks after the interest of countries at the upper end, instead of discounting, it puts up the income levels exorbitantly in case of the poor countries. These two extreme situations teach us a lesson to select a poverty line with reasonable considerations. Apparently, the conversion with a selection of a poverty line at the level of e or 2.718 is too unrealistic as shown in column 7 of Table 5. While it shows discounted income at all levels, it is well understood that the inherent pattern of conversion inflates the relative contribution of income to human development at the lower end. When we move to the next scheme with a poverty line at the level of the world average income (US$ 10,082 PPP as in HDR 2011), we see that it discounts income for the richer countries almost in a similar fashion as Atkinson's formula does. But, it does not penalise the high income countries as severely as the latter does (as shown in column 11 of Table 5). However, the overall discount is severe at the upper end, as it recognises a small fraction of income only as a means to human development (e.g., 12% in case of Qatar as shown in column 10 of Table 5). When we look at the conversion below the poverty line ($y < y^*$), the exaggeration becomes prominent. So, working with the present UNDP method (Equations 3a-3c) or with any other equivalent ones (Equations 7a, 7b and 7d) throughout the income range is subject to a serious limitation, as they inflate contribution of income to human development at the lower end to a great extent. Atkinson's formula is free from such limitation where indexing below poverty line is based on actual income. In line with the same, the new proposed method may too be adopted for all $y > y^*$ only, with $I = y$ for $y \leq y^*$ (as in Equations 7e-7f).

Table 5

Income Conversion under Different Schemes

Countries	GNI (US$ PPP)	Income Conversion under the New Proposed Scheme with Various Poverty Lines								Atkinson's Formula	
		I_1	(3)/(2)	I_2	(5)/(2)	I_3	(7)/(2)	I_4	(9)/(2)	W(y)	(11)/(2)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Qatar	107721	550	0.01	107721	1.00	31.50	0.00	12673	0.12	10573	0.10
Luxembourg	50557	514	0.01	100689	1.99	29.44	0.00	11845	0.23	10457	0.21
Japan	32295	493	0.02	96522	2.99	28.22	0.00	11355	0.35	10414	0.32
Italy	26484	484	0.02	94678	3.57	27.68	0.00	11138	0.42	10416	0.39
Russian Federation	14561	455	0.03	89117	6.12	26.06	0.00	10484	0.72	10283	0.71
Brazil	10162	438	0.04	85773	8.44	25.08	0.00	10091	0.99	10283	1.01
World Average	*10082*	*438*	*0.04*	*85700*	*8.50*	*25.06*	*0.00*	*10082*	*1.00*	*10082*	*1.00*
China	7476	424	0.06	82919	11.09	24.25	0.00	9755	1.30	7476	1.00
India	3468	387	0.11	75779	21.85	22.16	0.01	8915	2.57	3468	1.00
Nigeria	2069	363	0.18	70977	34.30	20.75	0.01	8350	4.04	2069	1.00
Nepal	1160	335	0.29	65598	56.55	19.18	0.02	7717	6.65	1160	1.00
Malawi	753	315	0.42	61580	81.78	18.01	0.02	7245	9.62	753	1.00
Eritrea	536	298	0.56	58420	108.99	17.08	0.03	6873	12.82	536	1.00
Liberia	265	265	1.00	51872	195.74	15.17	0.06	6102	23.03	265	1.00

Note: I_1: y* = 265 (GNI in Liberia); I_2: y* = 1,07,721 (GNI in Qatar); I_3: y* = e or 2.718; I_4: y* = 10,082 (World average GNI). y* = 10,082 in Atkinson's formula.

Source: HDR 2011 for GNI, self-elaboration otherwise.

In order to add more clarity, we display the index of income (as mentioned at the end of the previous section) for some selected countries in Table 6. In addition to this, we also present index of income according to Atkinson's formula (Equations 2a-2c, etc.) with usual minimum and maximum values of income as used in HDR 2011. The results for all the 187 countries are plotted in Figure 1 against GNI. We first adopt the new proposed method for the whole range of income (Equation 7d) and then for $y > y^*$ (Equations 7e-7f).

In Table 6 and Figure 1, we see that when applied for the whole range of income, the results from the new proposed method coincide with those corresponding to the present UNDP one. When it is applied for $y > y^*$, the line looks better than that of Atkinson's one,

as the curve looks more smooth and where asymptote starts quite late. However, in this case (in case of Atkinson's formula too), indexing below the poverty line ($y < y^*$) is based on the actual income depicting the true picture. So, visibly the new proposed method works with robustness and more clarity than the others.

Table 6

Index of Income under Various Methods

Countries	Index of income[§]			
	Present UNDP method	*New Proposed method (for all y)*	*New Proposed method (for y > y*)*	*Atkinson's formula*
(1)	*(2)*	*(3)*	*(4)*	*(5)*
Qatar	1.000	1.000	1.000	1.000
Luxembourg	0.892	0.892	0.934	0.989
Japan	0.827	0.827	0.895	0.985
Italy	0.799	0.799	0.878	0.985
Russian Federation	0.713	0.713	0.826	0.972
Brazil	0.662	0.662	0.795	0.972
China	0.618	0.618	0.587	0.704
India	0.508	0.508	0.268	0.322
Nigeria	0.434	0.434	0.157	0.188
Nepal	0.351	0.351	0.084	0.101
Malawi	0.289	0.289	0.052	0.062
Eritrea	0.240	0.240	0.035	0.042
Liberia	0.140	0.140	0.013	0.016

Note: § Conversions in columns 2, 3 and 5 are done as per columns 7, 9 and 11 of Table 5 respectively. Results in column 4 are based on conversion as per (7e) and (7f). Index of income (in all cases): (actual value – minimum value) / (maximum value – minimum value).

Source: Self-elaboration.

Figure 1

Income Conversions under Different Formulae
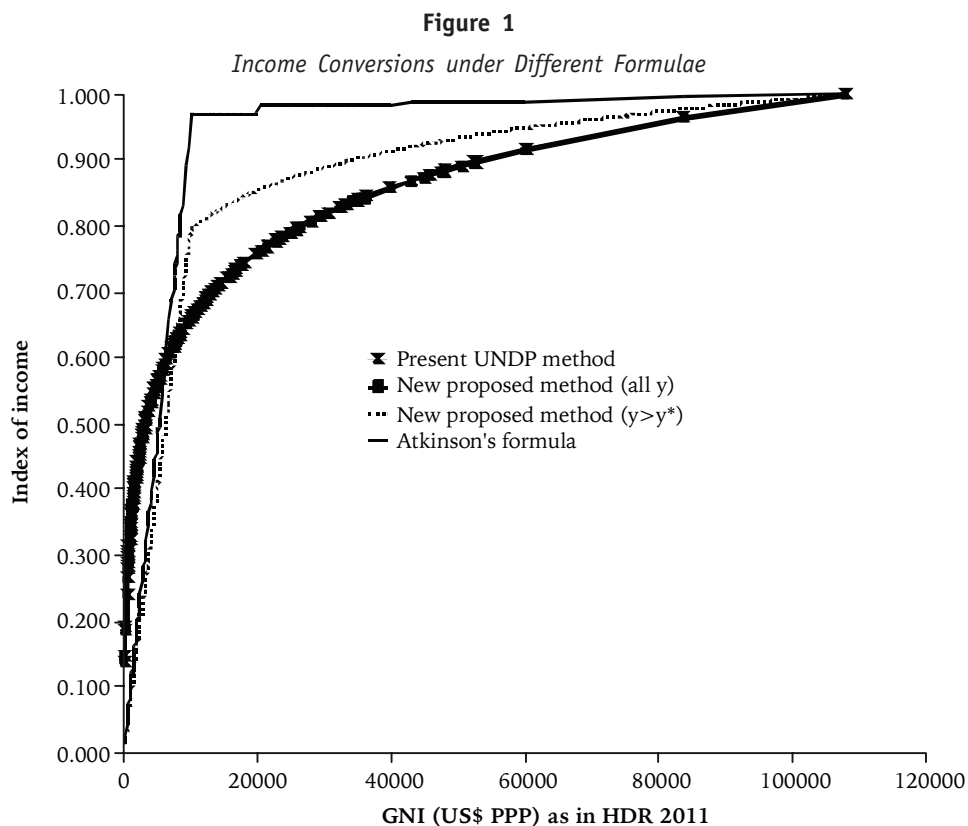
In the introductory section, we justified the need for knowing the extent of discounts. We see that it is not as extensive as the similar previous formula does. Also we are now in a position to comment that column 9 of Table 5, rather than column 2 of the same, is closer to column 2 or 3 of Table 6. This gives us an idea about how much income (relative to a poverty line) is actually necessary to maintain different standards of living in different countries. The difference between the figures in columns 2 and 9 for $y > y^*$ in Table 5 gives us a new thought to study the roles and abilities of the rich countries towards disbursing aid or else for the well-being of the others.

VIII. Conclusion

While converting income in the process of computing index of income in HDI or else, one may work with the proposed new method, which not only works with the spirit of Atkinson's formula, but also embodies merits of the older and newer ones associated with the HDI methodology so far.

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